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OPTIMAL FISCAL FEEDBACK ON DEBT IN AN ECONOMY WITH NOMINAL RIGIDITIES

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Optimal Fiscal Feedback on Debt in an Economy with Nominal Rigidities *

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Abstract

We examine the impact of different degrees of fiscal feedback on debt in an economy with nominal rigidities where monetary policy is optimal. We look at the extent to which different degrees of fiscal feedback enhances or detracts from the ability of the monetary authorities to stabilise output and inflation. Using an objective function derived from utility, we find the optimal level of fiscal feedback to be small. There is a clear discontinuity in the behaviour of monetary policy and welfare either side of this optimal level. As the extent of fiscal feedback increases, optimal monetary policy becomes less active because fiscal feedback tends to deflate inflationary shocks. However this fiscal stabilisation is less efficient than monetary policy, and so welfare declines. In contrast, if fiscal feedback falls below some critical value, either the model becomes indeterminate, or optimal monetary policy becomes strongly passive, and this passive monetary policy leads to a sharp deterioration in welfare.

Key Words: Fiscal Policy, Feedback Rules, Debt, Macroeconomic Stabilisation
JEL Reference Number: E52, E61, E63, F41

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1 Introduction

With the occasional and notable exception, most governments now see one of their primary economic responsibilities as ensuring that the national debt stays within reasonable bounds. In some cases explicit targets for the debt to GDP ratio have been announced, with the implication that if debt deviates from this target, some form of 'fiscal feedback' via taxes or spending will operate. However it is also recognized that any attempt to control the debt stock, or the public sector deficit, too tightly may induce instability in other macroeconomic variables. There is a trade-off between ensuring intergenerational equity through fiscal responsibility and the goal of short term macroeconomic stabilisation.

In this paper we examine this trade-off. Recently some studies (Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004) in particular) have looked at jointly optimal monetary and fiscal policies. While examining jointly optimal policy is of interest, and we compute such policies, it appears unrealistic given current institutional arrangements. It is generally the case that fiscal policy is far less flexible than monetary policy. Partly as a result, the focus of policy makers seems to be on how quickly any debt disequilibrium should be corrected, as the debates around the Stability and Growth Pact of the European Monetary Union illustrate.[1]

We assume that this 'fiscal feedback' from debt takes a very simple form, such that spending is adjusted by some fixed proportion of debt disequilibrium, and we examine the implications of alternative values for this feedback parameter. We compare this simple fiscal feedback to a fully optimal fiscal policy, where spending can also adjust directly to shocks. Our use of spending rather than taxes as the means of debt control allows us to abstract from issues of tax smoothing. We look at a closed economy, where monetary policy is determined optimally, assuming commitment. We compute welfare using an objective function derived from consumer utility.

Traditionally it was thought that some minimum level of fiscal feedback was required for a stable and determinate economy. However, the Fiscal Theory of the Price Level literature has argued that a determinate equilibrium may be possible when no feedback occurs, but where prices adjust to ensure the government's intertemporal budget constraint holds. (See Woodford (1996), but also Buiter (2002) for a more critical view.) However there have been few studies that have explicitly compared welfare under such a regime to a more conventional regime where sufficient fiscal feedback occurs. In this paper we make such a comparison, and find that determinate regimes involving little or no fiscal feedback are always welfare dominated by regimes with stronger fiscal feedback, essentially because

[1] This does not imply fiscal policy makers are 'irrational', but may simply reflect overriding political economy concerns that are outside the scope of this paper.

optimal monetary policy in regimes with little or no fiscal feedback is strongly 'passive' (in the sense of Leeper (1991)). We show that, in conventional regimes where sufficient fiscal feedback occurs to allow an active monetary policy, then the optimal amount of fiscal feedback is modest. This optimal degree of fiscal feedback leaves debt following a stochastic process close to (but not exactly equal to) a random walk. We also show that, in between these two regimes, there are values of fiscal feedback which leave the economy indeterminate.

The paper is organised as follows. Section 2 outlines our core model, which embodies nominal rigidity through Calvo contracts, and where consumers are 'infinitely lived'.We discuss the solution method in Section 3 and examine the implications of the model for both the optimal degree of fiscal feedback, and for fully optimal fiscal policy in Section 4. Section 5 outlines a variant to our model, where consumers are of the Blanchard Yaari type. Although we find that our results are not changed significantly in a quantitative way, we do note some important qualitative differences to fully optimal fiscal policy in particular. Section 6 concludes.

2 The Model

Our model is of a closed economy, with government, where government spending involves producing public goods that are valued by consumers. Consumers also value private consumption and leisure and are infinitely lived. Firms set prices subject to Calvo contracts. Monetary policy is determined optimally under commitment to maximise a social welfare function derived from consumer's utility. Around the steady state the fiscal authority varies government spending in proportion to debt disequilibrium, although this 'fiscal feedback' may be zero.

2.1 Consumers

Our model of the household sector is familiar from Woodford (2003). Our economy is inhabited by a large number of individuals, who specialize in the production of a differentiated good (indexed by z), and who spend $h(z)$ of effort in its production. They consume a basket of goods C, and derive utility from per capita government consumption G. Individuals' maximization problem is

$$\max_{\{C_v,h_v\}_{v=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} \beta^{v-t} \left[u(C_v) + f(G_v) - v(h_v(z)) \right]. \tag{1}$$

The price of a differentiated good z is denoted by $p(z)$, and the aggregate price level is P.[2] An individual chooses optimal consumption and work effort to maximise criterion (1) subject to the

[2] Adding taste shocks along the lines of Woodford (2003) would have no material impact on our results.

demand system and the intertemporal budget constraint:

$$P_tC_t + \mathcal{E}_t\left(Q_{t,t+1}\mathcal{A}_{t+1}\right) \leq \mathcal{A}_t + (1-\tau)\left(w_t(z)h_t(z) + \Pi_t(z)\right) + T_t, \tag{2}$$

where $P_tC_t = \int_0^1 p(z)c(z)dz$ is nominal consumption, $\mathcal{A}_t$ are nominal financial assets of a household, Π_t is profit and T_t is a lump-sum subsidy. Here w is the wage rate, and τ is a tax rate on income. $Q_{t,t+1}$ is the stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent amount $\mathcal{A}_{t+1}$ of wealth into period $t+1$. The riskless short term nominal interest rate i_t has the following representation in terms of the stochastic discount factor:

$$\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1+i_t)}.$$

Each individual consumes the same basket of goods. Goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by $\epsilon_t > 1$ (which is a stochastic elasticity with mean ϵ[3]), $C_t = \left[\int_0^1 c_t^{\frac{\epsilon_t - 1}{\epsilon_t}}(z)dz\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$.

We assume no Ponzi schemes and that the net present value of individual's future income is bounded. We also assume that the nominal interest rate is positive at all times. These assumptions rule out infinite consumption and allow us to replace the infinite sequence of flow budget constraints of the individual by a single intertemporal constraint,

$$\mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}C_vP_v \leq \mathcal{A}_t + \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}\left\{(1-\tau)\left(w_v(z)h_v(z) + \Pi_v(z)\right)) + T_v\right\}. \tag{3}$$

The optimisation requires that the household exhaust its intertemporal budget constraint and, in addition, the household's wealth accumulation must satisfy the transversality condition

$$\lim_{s\to\infty} \mathcal{E}_t\left(Q_{t,s}\mathcal{A}_s\right) = 0. \tag{4}$$

We assume the specific functional form for the utility from consumption component, $u(C_v) = \frac{C_v^{1-1/\sigma}}{1-1/\sigma}$.[4] Household optimisation leads to the following dynamic relationship for aggregate consumption:

$$C_t = \mathcal{E}_t\left(\left(\frac{1}{\beta}\frac{P_{t+1}}{P_t}Q_{t,t+1}\right)^{\sigma} C_{t+1}\right) \tag{5}$$

where parameter σ is defined as $\sigma = -u_C(C)/u_{CC}(C)C$. Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{t+1} = (1+i_t)\left(\mathcal{A}_t + (1-\tau)P_tY_t - P_tC_t\right) \tag{6}$$

[3] We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

[4] We do not need to assume this specific functional forn of utility for our results in this section, but it is required when we introduce Blanchard-Yaari consumers.

We define $A_t = \mathcal{A}_t/P_{t-1}$ and linearise equations (5) and (6) around the steady state (for each variable X_t with steady state value X, we use the notation $\hat{X}_t = \ln(X_t/X)$). Equation (5) leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_t = \mathcal{E}_t\hat{C}_{t+1} - \sigma\left(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}\right) \tag{7}$$

Inflation is $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and we assume inflation is zero in equilibrium.

The assets equation can be linearised as

$$\hat{A}_{t+1} = \hat{\imath}_t + \frac{1}{\beta}\left(\hat{A}_t - \hat{\pi}_t + \frac{(1-\tau)}{A}\hat{Y}_t - \frac{\rho}{A}\hat{C}_t\right).$$

Here $\rho = C/Y$ is the steady state share of private consumption in Y and A is the steady state level of real assets as a share of Y.

2.2 Price Setting

Price setting is based on Calvo contracting as set out in Woodford (2003). Each period agents recalculate their prices with fixed probability $1-\gamma$. If prices are not recalculated (with probability γ), they remain fixed. Following Woodford (2003) and allowing for government consumption terms in the utility function, we can derive the following Phillips curve for our economy[5]:

$$\hat{\pi}_t = \beta\mathcal{E}_t\hat{\pi}_{t+1} + \kappa_c\hat{C}_t + \kappa_y\hat{Y}_t + \hat{\mu}_t \tag{8}$$

where the shock $\hat{\mu}_t$ is a mark-up shock . Although the *constant* income tax τ has no effect on the dynamic equations for log-deviations from the flexible price equilibrium, it alters the equilibrium choice between consumption and leisure for the consumer. The coefficients of the Phillips curve are:

$$\kappa_c = \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma\left(\psi+\epsilon\right)\sigma}, \quad \kappa_y = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\left(\psi+\epsilon\right)}.$$

where $\psi = v_y/v_{yy}y$.

Under flexible prices and in the steady state the real wage is always equal to the monopolistic mark-up $\mu_t = -(1-\epsilon_t)/\epsilon_t$. Optimisation by consumers then implies (we assume the production function $y_t = h_t$) :

$$\frac{\mu^w\left(1-\tau\right)}{\mu_t} = \frac{v_y(y_t^n(z))}{u_C(C_t^n)} \tag{9}$$

where μ^w is a steady state employment subsidy, which we discuss below.

[5]The derivation is identical to the one in Woodford (2003), amended by the introduction of mark-up shocks as in Beetsma and Jensen (2004a).

2.3 Fiscal Authorities

The government buys goods (G), taxes income (with tax rate τ), and issues nominal debt $\mathcal{B}$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{t+1} = (1 + i_t)(\mathcal{B}_t + P_t G_t - \tau P_t Y_t)$$

We assume that the tax rate on income is fixed. This equation can be linearised as (defining $B_t = \mathcal{B}_t / P_{t-1}$ and denoting the steady state ratio of debt to output as B) :

$$\hat{B}_{t+1} = \hat{\imath}_t + \frac{1}{\beta}\left(\hat{B}_t - \hat{\pi}_t + \frac{1-\rho}{B}\hat{G}_t - \frac{\tau}{B}\hat{Y}_t\right) \tag{10}$$

We postulate that disequilibrium in government spending is related to debt disequilibrium according to the following simple feedback rule:

$$\hat{G}_t = -\lambda \hat{B}_t \tag{11}$$

Our reason for adopting a simple mechanistic rule for fiscal policy is that it more accurately reflects institutional rigidities in policymaking. Of course there are a variety of potential simple rules, but rule (11) focuses directly on the concern with debt disequilibrium, and has been widely used in the literature. We also compute fully optimal fiscal policies, such that monetary and fiscal policy are jointly determined in an optimal manner under commitment, along lines discussed below. We show below that, in the context of the particular model we use and with a fully optimal monetary policy, the loss in welfare generated by adopting rule (11) rather than a fully optimal policy is small.[6]

Government expenditures constitute part of demand

$$Y_t = C_t + G_t \tag{12}$$

and in steady state $G = (1-\rho)Y$. The linearised aggregate demand equation is then:

$$\hat{Y}_t = (1-\rho)\hat{G}_t + \rho\hat{C}_t. \tag{13}$$

2.4 Behaviour of the Economy

We now write down the final system of equations for the 'law of motion' of the out-of-steady-state economy. We simplify notation by using lower case letters to denote 'gap' variables, where the gap is

[6] If monetray policy is constrained or absent, as in the case of a monetary union member facing asymmetric shocks for example, then fiscal policy may well have a useful role to play in stabilisation: see Kirsanova, Satchi, Vines, and Wren-Lewis (2007) for example.

the difference between actual levels and natural levels i.e. $x_t = \hat{X}_t - \hat{X}_t^n$. We omit the expectational superscript, assuming rational expectations, $\mathcal{E}_t X_{t+1} = X_{t+1}$ for any variable X.

$$c_t = c_{t+1} - \sigma(i_t - \pi_{t+1}) \tag{14}$$

$$\pi_t = \beta\pi_{t+1} + \kappa_c c_t + \kappa_y y_t + \hat{\mu}_t \tag{15}$$

$$y_t = (1-\rho)g_t + \rho c_t \tag{16}$$

$$a_{t+1} = i_t + \frac{1}{\beta}\left(a_t - \pi_t + \frac{(1-\tau)}{A}y_t - \frac{\theta}{A}c_t\right) \tag{17}$$

$$g_t = -\lambda b_t \tag{18}$$

$$b_t = a_t \tag{19}$$

The model consists of an intertemporal IS curve (14), the Phillips curve (15), an aggregate demand equation (16), and an equation explaining the evolution of assets (17). The fiscal feedback rule is given by (18). We could use the debt accumulation equation (10) instead of (17) as they are equivalent (equation (19)).

This system describes the dynamic behaviour of the economy as observed by a policymaker. Equations (14) and (15) describe the reaction function of the private sector. As we discussed above, the private sector chooses consumption and inflation at each period in time, such that their future utility and profits are maximised, given the evolution of state variables and policy. The fiscal authorities are a non-strategic player in this set-up, as they mechanically react to the level of domestic debt. Therefore, the level of assets (debt), government expenditures and output are predetermined state variables, and the interest rate is the policy variable.

2.5 Central Bank's decisions

We assume that the central bank explicitly maximises the aggregate utility function:

$$\max_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left[u(C_s) + f(G_s) - \int_0^1 v(h_s(z))dz\right]. \tag{20}$$

We show in Appendix D.2 that (20) implies the following loss function

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left[a_\pi \pi_s^2 + a_c c_s^2 + a_g g_s^2 + a_y y_s^2\right] + \mathcal{O}(3) \tag{21}$$

where $\mathcal{O}(3)$ collects terms of higher than second order and terms independent of policy, and all a_i are positive. This quadratic approximation to social welfare is obtained assuming that there is a production subsidy μ^w that eliminates the distortion caused by monopolistic competition and income taxes. (This follows Woodford (2003). Sutherland (2002) and Benigno and Woodford (2004) use an alternative way of eliminating first order terms from welfare, while Schmitt-Grohe and Uribe (2004) do not use a linear quadratic framework, but instead use a Ramsey approach.) We assume that the central bank has sufficient credibility to commit to the time inconsistent plan – in other words it can implement the first best time inconsistent solution.

Note also that expression (21) contains a quadratic term in government spending, g. This term enters the welfare expression partly because it is assumed in (1) that households derive utility from consumption of public goods, and that the level of government spending in steady state reflects this. However, if we instead assumed that consumers derived no utility from government spending, and that government spending was pure waste, a quadratic term in g would still appear in social welfare through the national income identity.

3 Model Solution

The central bank chooses the interest rate to minimise social loss (21) subject to the reaction of the private sector (14), (15) and the evolution of predetermined state variables (17)-(19).

Let the period at which optimisation is taking place be period t. We construct the Lagrangian

$$w = \min_{\{i_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} H_s$$

where each term has the following form

$$\begin{aligned} H_s \;=\;& \frac{1}{2}\beta^{s-t}\left[a_\pi \pi_s^2 + a_c c_s^2 + a_g \lambda^2 b_s^2 + a_y\left(-(1-\rho)\lambda b_s + \rho c_s\right)^2\right] \\ & +\beta^{s-t} L_{s+1}^c \left(c_{s+1} - \sigma(i_s - \pi_{s+1}) - c_s\right) \\ & +\beta^{s-t} L_{s+1}^\pi \left(\beta\pi_{s+1} + (\kappa_c + \kappa_y \rho)\, c_s - \kappa_y(1-\rho)\lambda b_s + \hat{\mu}_s - \pi_s\right) \\ & +\beta^{s-t} L_{s+1}^b \left(i_s + \frac{1}{\beta}\left(\left(1 - \frac{(1-\tau)(1-\rho)\lambda}{B}\right) b_s - \pi_s - \frac{\tau\rho}{B} c_s\right) - b_{s+1}\right) \end{aligned}$$

In order to minimise the loss function, we differentiate the Lagrangian with respect to L^c, L^π, L^b,

π, c, b and i. The first order conditions for optimality are:

$$\frac{\partial w}{\partial \pi_s} = 0 = \beta^{s-t} a_\pi \pi_s + \sigma \beta^{s-t-1} L_s^c + \beta^{s-t+1} L_s^\pi - \beta^{s-t} L_{s+1}^\pi - \beta^{s-t-1} L_{s+1}^b \tag{22}$$

$$\frac{\partial w}{\partial c_s} = 0 = \beta^{s-t} a_c c_s + \beta^{s-t} a_y \rho \left(-(1-\rho)\lambda b_s + \rho c_s\right) + \beta^{s-t-1} L_s^c - \beta^{s-t} L_{s+1}^c \tag{23}$$
$$+ \left(\kappa_c + \kappa_y \rho\right) \beta^{s-t} L_{s+1}^\pi - \frac{\tau\rho}{\beta B} \beta^{s-t} L_{s+1}^b$$

$$\frac{\partial w}{\partial b_s} = 0 = \beta^{s-t} a_g \lambda^2 b_s - (1-\rho)\lambda \beta^{s-t} a_y \left(-(1-\rho)\lambda b_s + \rho c_s\right) \tag{24}$$
$$-\kappa_y (1-\rho)\lambda \beta^{s-t} L_{s+1}^\pi + \left(1 - \frac{(1-\tau)(1-\rho)\lambda}{B}\right) \beta^{s-t-1} L_{s+1}^b - \beta^{s-t-1} L_s^b$$

$$\frac{\partial w}{\partial i_s} = 0 = -\sigma \beta^{s-t} L_{s+1}^c + \beta^{s-t} L_{s+1}^b \tag{25}$$

$$\frac{\partial w}{\partial L_{s+1}^c} = 0 = c_{s+1} - \sigma(i_s - \pi_{s+1}) - c_s \tag{26}$$

$$\frac{\partial w}{\partial L_{s+1}^\pi} = 0 = \beta \pi_{s+1} + \left(\kappa_c + \kappa_y \rho\right) c_s - \kappa_y (1-\rho)\lambda b_s + \hat{\mu}_s - \pi_s \tag{27}$$

$$\frac{\partial w}{\partial L_{s+1}^b} = 0 = i_s + \frac{1}{\beta}\left(\left(1 - \frac{(1-\tau)(1-\rho)\lambda}{B}\right) b_s - \pi_s - \frac{\tau\rho}{B} c_s\right) - b_{s+1} \tag{28}$$

along with initial conditions $L_t^c = L_t^\pi = 0, b_t = \bar{b}$, see Currie and Levine (1993).[7]

We discuss the solution to this problem in detail in Appendix A. In particular, we note the relationship between the number of jump variables in the system and the size of the system's eigenvalues, which governs whether the system is stable, unstable or indeterminate. We show how the size of key eigenvalues depends on the extent of fiscal feedback λ. These results are summarised in Section 4 below. In what follows, we also use the fact that the solution for the optimal interest rate rule can be presented in the form of linear relationship (see Appendix B):

$$i_t = \theta_\mu \hat{\mu} + \theta_b b_t + \vartheta_{L\pi} L_t^\pi + \vartheta_{Lc} L_t^c \tag{29}$$

with feedback coefficients θ on predetermined states and with feedback coefficients ϑ on predetermined Lagrange multipliers.

4 Optimal Monetary Policy and Welfare

Our focus of interest is in exploring the implications of different values of the fiscal feedback parameter λ. We are interested in two key questions. First, can we distinguish clearly between two policy 'regimes', as suggested by the Fiscal Theory of the Price Level and the results in Leeper (1991) and

[7] In all the analysis below we take $\bar{b} = 0$ i.e we assume we start from a position in which debt is at its steady state.

Leith and Wren-Lewis (2000)? If we can, how does welfare and optimal monetary policy compare between regimes? Second, what is the optimal degree of fiscal feedback on debt, and what are the implications for welfare and monetary policy of departing from this optimum?

The calibration of the model is discussed in Appendix C. Figure 1 presents the values of some key magnitudes as we change the degree of fiscal feedback λ. The left and right panels are identical except for the scale of λ: the left hand side focuses on small values of fiscal feedback above zero, whereas the right hand panel gives results for a much broader range. The bottom charts plot the loss that results from a 1% cost push shock expressed as percent reduction in steady-state consumption. The charts above plot parameters θ from the implied reaction function for monetary policy (29): i.e. the feedback coefficient on the mark-up shock, θ_μ (top panel), and on debt, θ_b.[8].

4.1 The two policy regimes

Figure 1 and the analysis in Appendix A suggest that there are two determinate regimes, in each of which all processes are either stationary or unit root. We discuss each of these two regimes first, and then consider the 'gap' between them.

The first regime occurs when $\lambda = 0$. Here we have no feedback from debt disequilibrium to fiscal variables, so we might suppose that debt in this model would be unstable. However, the results show that monetary policy is able to stabilise the system. Our system has four predetermined variables, and we report in Appendix A that for $\lambda = 0$ the system has exactly four eigenvalues which are less than or equal to one in absolute value, and all other eigenvalues are not smaller than $1/\beta$. Monetary policy achieves stability in two ways, both shown in Figure 1. First, any positive debt disequilibrium leads to a large fall in interest rates, which leads to correction through the government's budget constraint. Second, the reaction to a positive cost-push shock is also to reduce interest rates. As a positive cost-push shock also raises debt, this reduction in interest rates also helps achieve debt stability.

In contrast, in the second regime, where $\lambda \geq \lambda^*$ (we discuss the value of λ^* below), the reaction to a cost-push shock is to raise interest rates. The system is again determinate: we report in Appendix A, that for $\lambda \geq \lambda^*$ the system has exactly four eigenvalues which are less than or equal to one in absolute value, and all other eigenvalues are not smaller than $1/\beta$. The difference in response to an inflationary shock in the two regimes reflects the distinction between 'active' and 'passive' regimes identified in Leeper (1991) and Leith and Wren-Lewis (2000), although their analysis only considered very simple rules for monetary policy rather than optimal monetary policy. Leeper (1991) describes an active

[8] As formula 29 shows, there will also be feedback on predetermined Lagrange multipliers ϑ_L. We do not plot them as they are less informative: ϑ_L represent the integral control part of the reaction function and therefore feedback on slow moving variables.

monetary policy regime as a coefficient on inflation in a Taylor type rule for nominal interest rates greater than unity (i.e. the 'Taylor principle' is satisfied). In Leeper (1991) and Leith and Wren-Lewis (2000) active monetary policy is associated with non-negligible fiscal feedback.

In the area between these two points, $0 < \lambda < \lambda^*$ the system has three eigenvalues which are strictly inside the unit circle, two eigenvalues that are outside the unit circle but strictly less than $1/\beta$, and all other eigenvalues are not smaller than $1/\beta$ (see Appendix A). Although the transversality conditions imply that debt grows at less than the real rate of interest, and the total cost of such dynamics will be finite (because of discounting), in this case there are more eigenvalues which are less than $1/\beta$ in absolute value than the number of predetermined variables. As a result, there is a continuum of such 'moderately' explosive solutions, so we cannot plot anything for $0 < \lambda < \lambda^*$ in Figure 1. At the boundaries of the region $\lambda = 0$ and $\lambda = \lambda^*$, the size of one of these unstable roots falls to unity, and so we have a determinate system with random walk properties.

There is a clear parallel between our results using optimal monetary policy and the active and passive regimes described by Leeper (1991) and Leith and Wren-Lewis (2000). [9] In their case a passive monetary policy is defined as a negative response of real interest rates when inflation is above target in a Taylor rule, whereas in our case it corresponds to a fall in real interest rates following a positive cost-push shock. When there is no fiscal feedback, fiscal instruments do nothing to prevent a debt interest spiral. To avoid an explosive solution for debt, monetary rather than fiscal policy must stabilise the government's debt stock. Following a cost push shock, output will fall, lowering tax receipts and raising debt. To avoid a debt interest spiral, real rates must also fall, so that the governments intertemporal budget constraint continues to hold. This is illustrated in Figure 3, which plots the impulse response for key economic variables for the two values of $\lambda, \lambda = 0$ as a dashed line, and $\lambda = \lambda^{\min} > \lambda^*$ as a solid line, following a cost push shock, where $\lambda^{\min}$ delivers the minimum loss.

Our finding of a region with a continuum of solutions for some (small) values of λ is also of interest. In certain respect, such solutions share some properties of *indeterminate* solutions discussed in the Real Business cycles literature. There, indeterminacy is a common feature of (rational expectations) economies that exhibit some market imperfections. It typically arises in markets with external effects or with monopolistic competition. In our model we have a distortion generated by price rigidity, but we have offset monopolistic distortions with a subsidy. In our case multiplicity (which for convenience we will refer to as indeterminacy) is clearly linked with the presence of government debt, and the

[9] The use of a Taylor rule to describe monetary policy allows Leith and Wren-Lewis (2000) to calculate analytically a critical value for λ that divides the two policy regimes from the determinate stability condition (which is a necessary but not sufficient condition for a determinate or stable solution). They find that this critical value of λ is small, in the sense of being of the same order of magnitude as the steady state value of real interest rates, and we obtain a similar result for λ^* as we note below.

problem that uncontrolled debt poses for monetary policy. As our two determinate policy regimes demonstrate, monetary policy has to behave in a very different way if it is trying to control debt rather than control inflation, and in between these two regimes the rational expectations private sector has a continuum of responses consistent with the two different incentives of the monetary authorities. Note however that under any of these continuum of responses of the private sector, the economy exhibits a 'controlled explosion', i.e. the rate of divergence is strictly less than the real rate of interest. The monetary policy that tries to achieve two targets is unable to ensure the convergence of the economy back to the steady state.

4.2 Optimal value of the fiscal feedback

A key result from Figure 1 is that the 'passive' regime is less successful at stabilising shocks than an active policy, whatever the value of λ in the range $\lambda \geq \lambda^*$. This is perhaps not surprising, but we are not aware that this has been formally demonstrated in the literature before. While the papers cited above, and the Fiscal Theory of the Price Level more generally, have shown that a lack of fiscal feedback does not necessary lead to model indeterminacy/instability, it is clear from our results that an active monetary policy regime is superior at stabilising the economy, assuming that monetary policy is optimal in both regimes.

Turning to the determinate regime where there is fiscal feedback ($\lambda \geq \lambda^*$), the optimal value of λ (which we denote as $\lambda^{\min}$) is very close to the lowest possible value that sustains this regime, λ^*(see Table 1). As we noted above, at $\lambda = \lambda^*$, one of the system's eigenvalues is exactly unity, and this corresponds with a unit root process for debt At the optimal value of lambda ($\lambda = \lambda^{\min}$), therefore, debt is *almost* a unit root process, but will eventually return to its original steady state value. This is of particular interest in the light of results in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004), that suggest that a completely optimal fiscal policy would exhibit a pure 'random walk' for debt. We can confirm this result for our model: if we compute a fully optimal fiscal policy (so that both monetary and fiscal policies determined cooperatively are optimal under commitment), debt follows a exact random walk[10]. This is illustrated in Fig 4, where we plot the paths of government spending and debt under a fully optimal fiscal policy, and in the case of $\lambda = \lambda^{\min}$.

The intuition behind the random walk result is as follows. Leaving debt permanently higher has a welfare cost, which is that government spending must be permanently below the optimal level implied by agents' preferences for public goods, in order to 'finance' the additional debt. However

[10] One notable difference between our analysis and those in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004) is that we treat government spending, rather than taxes, as the fiscal policy instrument.

with discounting this permanent cost is finite, and the departure from optimal government spending is small. To return debt to its original steady state level quickly would require much larger cuts in government spending in the short term. Although the intertemporal government budget constraint roughly equates the monetary value of these alternative paths, the fact that welfare includes quadratic terms in spending means that smooth paths tend to be preferred to others (as in tax smoothing, for example), and so the random walk result is less costly than larger, short term adjustment. However, note from Figure 4 that under fully optimal fiscal policy the debt implications of the shock are not completely accommodated: there is an attempt in the first period to reduce spending and thereby moderate the eventual increase in debt. Leith and Wren-Lewis (2006) analytically confirm that this will always be the case, and discuss the reasons for it. This initial path for government spending cannot be replicated under our simple feedback rule, because spending is tied to debt. Although this short term difference is small in quantitative terms, it helps explain why the optimal level of fiscal feedback ($\lambda = \lambda^{\min}$) is very slightly above that required for a pure random walk.

Table 1 also shows that the fully optimal fiscal policy, the 'random walk' level of fiscal feedback ($\lambda = \lambda^{*}$) and the optimal level of fiscal feedback ($\lambda = \lambda^{\min}$) are all associated with an active monetary policy, in the sense that interest rates rise following a cost-push shock (see also figures 1 and 3).[11]

The optimal value of fiscal feedback, although it does not produce a pure random walk, is extremely close to a random walk. In Figure 4 we note that debt is substantially above its original level even after 500 years. The value $\lambda = \lambda^{\min}$ implies that for every \$100 of debt disequilibrium, government spending is reduced by \$1.25 dollars a quarter. This value is close to the value of the steady state real interest rate, as we would expect given the near random walk behaviour of debt. (It is not identical to this level, because changes in government spending have implications for other variables including labour supply and output, and this in turn influences tax receipts.) For $\lambda > \lambda^{\min}$, Figure 1 shows that the loss function steadily increases, although even when adjustment becomes very large (a value of $\lambda = 1$ implies that government spending falls by over \$40 each quarter for every \$100 in debt disequilibrium), the loss is never as great as in the case of $\lambda = 0$. However, the increase in loss does demonstrate the macroeconomic costs involved in attempting to correct debt disequilibrium quickly.[12]

[11] In this respect our results can be compared with those in Schmitt-Grohe and Uribe (2004). Their approach does not allow the identification of a feedback rule for monetary policy, so instead they employ a regression technique to examine the nature of optimal monetary policy. Schmitt-Grohe and Uribe (2004) estimate a relationship between inflation and interest rates, and they find the coefficient on inflation is negative. However, it is not clear that this indicates a passive policy. Schmitt-Grohe and Uribe (2004) also show that optimal policy involves an almost constant inflation rate, which appears inconsistent with a passive monetary policy where inflation is used to correct the government's fiscal position. Our approach allows us to study this question in a direct way, by constructing an explicit policy reaction function.

[12] The costs of larger λ 'come from' the quadratic term in g in social welfare. If we artificially delete this term, the loss function after λ^{*} would be flat. However, as we note above, it would be illegitimate to delete this term in g, even if all

While a policy that set λ a little above $\lambda^{\min}$ would have little cost – and would also be prudent given our indeterminacy findings – setting a much larger value for λ would incur significant costs.

It is worth noting that increasing λ does have a noticeable impact on optimal monetary policy: the response of real interest rates to the cost push shock falls substantially as λ increases. The reason for this is as follows. For large λ, fiscal policy helps stabilise the impact of a cost push shock. The shock raises debt (see above), which with large λ implies a substantial decline in government spending. This deflates the economy, implying less of a need for real interest rates to rise. However, this form of feedback is less efficient at demand stabilisation than monetary policy, as the values for the welfare loss show. Although both fiscal and monetary policy act directly on demand (through public and private consumption respectively), fiscal policy only acts when debt changes, whereas optimal monetary policy can respond directly to inflationary shocks, and is therefore more efficient.

One final comparison of interest is to compare welfare under a fully optimal fiscal policy with welfare when fiscal feedback is optimal. We argued above that, given current institutional arrangements, fiscal feedback represents a more realistic view of fiscal policy setting than a fully optimal fiscal policy, but it is interesting to note what the costs of this are. Table 1 compares losses for simple fiscal feedback and a fiscal policy that is fully optimal i.e. like monetary policy, it responds directly to the cost push shock as well as debt. Losses under the optimal level of debt feedback are only slightly above those under a fully optimal fiscal policy. In this case, therefore, there is only a small cost in restricting fiscal policy to just respond to debt.

5 Blanchard-Yaari consumers

The results discussed so far assume that consumers are infinitely lived, so changes in government debt/personal wealth have no direct effect on the pattern of consumer spending over time. In this section we examine an alternative set up, where consumers have finite lives, using the framework due to Blanchard and Yaari (Blanchard (1985)). (Blanchard/Yaari consumers are also modelled in Leith and Wren-Lewis (2007) who examine issues of stability and monetary/fiscal policy interaction in a monetary union, as well as Smets and Wouters (2002) and Ganelli (2005)). With Blanchard/Yaari consumers, we now have a direct route whereby changes in government debt will influence changes in consumption, and we want to examine the extent to which the results described above continue to hold. Introducing Blanchard/Yaari consumers does, however, introduce costs in terms of complexity, which is why we do not examine them in the base case. We briefly outline changes to the model and

government spending was pure waste.

the welfare metric in this section, and give all details in an Additional Appendix[13].

5.1 The model

We need to make a number of changes to our model, described by equations (14)– (19). First, as consumers have a constant probability of death, p, the discount factor in formula (1) becomes $\beta/(1+p)$. Second, in the household budget constraint (2), the discount factor takes account of mortality, $\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1+i_t)(1+p)}$. Third, these modifications and the fact that we now have an infinite number of living cohorts at each moment of time, results in a new system for aggregate variables. The first order conditions for individual consumption, and then aggregation of all such behavioural equations, leads to a pair of equations for aggregate consumption and for the average propensity to consume, instead of the single Euler equation (7):

$$\hat{C}_t = [\beta(1+i)]^{-\sigma}(\mathcal{E}_t\hat{C}_{t+1} + \frac{p\rho}{\Phi\theta}(\mathcal{E}_t\hat{A}_{t+1} - \mathcal{E}_t\hat{\pi}_{t+1} - \mathcal{E}_t\hat{\Phi}_{t+1})) - \sigma(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}), \tag{30}$$

$$\frac{(1+p)(1+i)}{\beta^{\sigma}(1+i)^{\sigma}}\hat{\Phi}_t = \mathcal{E}_t\hat{\Phi}_{t+1} - (1-\sigma)(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}), \tag{31}$$

where $1/\Phi_t$ is average propensity to consume out of total resources, resources which consist of nominal financial wealth and human wealth. Equations (30) and (31) can be written in terms of gap variables. The resulting four equations should now be included in a system like that shown in equations (14)–(19), instead of equation (14).

To evaluate gains and losses we need a welfare metric. In the Blanchard-Yaari case, unlike in the infinitely-lived case, there is no obvious choice. Ideally total welfare should be evaluated using a social welfare function that aggregates across generations and weights the utility of every generation. It is not clear, however, how to treat future unborn generations. Calvo and Obstfeld (1988) discuss the importance of including unborn generation in the social welfare metric. If they are excluded, we introduce an additional source of time-inconsistency, as the policy which treats some particular generation differently will be necessarily time-inconsistent. However, straightforward aggregating of the utilities of unborn generations is not feasible for computational reasons. One way to overcome this difficulty is to suggest that the government uses a weighting scheme that makes the aggregate welfare of overlapping generations equivalent to the welfare of one infinitely long lived generation of consumers. A similar strategy was also adopted by Calvo and Obstfeld (1988). We therefore use formulae (21) to obtain our results.

[13] It is avalable at www.ex.ac.uk/˜tkirsano.html and upon request from the authors.

5.2 Results

Figure 2 shows results when we set mortality to a realistic, non-zero value: $p = 0.01$ corresponds to a working life of approximately 30 years. With Blanchard/Yaari consumers, the extent of fiscal feedback by policymakers will now impact on consumers as well as government spending. However, a number of the results noted above remain. We continue to obtain two determinate policy regimes. With negligible fiscal feedback, optimal monetary policy is still passive, in the sense that interest rates fall following a cost push shock. For $\lambda \geq \lambda^*$ (using the same notation as above), monetary policy is active, and this policy combination always dominates the negligible fiscal feedback regime in terms of welfare. Once again, the optimum value of fiscal feedback ($\lambda = \lambda^{\min}$) is only slightly higher than the minimum value in this active monetary policy area ($\lambda = \lambda^*$).

There are two differences introduced by Blanchard Yaari consumers. First, the range of λ over which the model is indeterminate shifts slightly to the right. This means that a determinate passive regime operates not just at $\lambda = 0$, but also for very small values of feedback just above zero (in fact, up to $\lambda = 0.0014$). It also means that the 'active' regime starts at a value of λ just above the value of λ^* identified for infinitely lived consumers. This is consistent with results in Leith and Wren-Lewis (2000), where the critical value of λ derived from the determinate stability condition is a positive function of p.[14] The economic reason for this is as follows. A cost push shock raises debt, and this has a positive impact on demand through consumption with Blanchard Yaari consumers. As a result, monetary policy will generate a larger increase in interest rates, which in turn requires a larger decrease in government spending to prevent a debt interest spiral. In fact, there is a natural neutrality result here. The net impact of debt on demand combines the positive wealth effect from Blanchard–Yaari consumers with the negative effect operating through fiscal feedback. It seems logical that if the former increases (because of larger p), then optimal λ should rise in a corresponding way, thereby neutralizing the overall impact of debt on demand.

We also compute the fully optimal fiscal policy when we have Blanchard Yaari consumers. Recall that with infinitely lived consumers, this policy implied a pure random walk for debt, a result that is consistent with findings in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004). However, there has until now been no equivalent analysis in a model where consumers have finite lives and there are no bequests. We find that the random walk result does not hold in this case. The fully

[14] Our results go beyond those in Leith and Wren-Lewis (2000), who also consider Blanchard–Yaari consumers, in three respects. First, we show for negligible fiscal feedback that the optimal monetary policy is still passive (it responds negatively to inflation) even though it can also feedback directly from debt. Second, we show that the optimal monetary policy is strongly passive: the negative feedback on the cost push shock and inflation is very large. (This result is hinted at, but not established, in Leith and Wren-Lewis (2000).) Third, Figure 2 shows that this passive monetary policy, while it stabilizes debt, has a clear welfare cost compared to the alternative regime with significant fiscal feedback.

optimal monetary and fiscal policy produce a system where one of the eigenvalues is very close to one, but not equal to one. The reason for this is as follows. In a model with Blanchard Yaari consumers, the steady state real interest rate is no longer always equal to the rate of time preference, but instead is increasing in the steady state level of debt. A standard result from consumption smoothing is that if the real rate of interest differs from the rate of time preference we get 'tilting', and the same applies in this case to the path of public consumption chosen when the policy maker optimises. This makes a pure random walk outcome suboptimal. However, as Figure 4 shows, the behaviour of debt, both for a fully optimal policy and for optimal fiscal feedback, are pretty close to a random walk, with less than half of debt disequilibrium eliminated after 250 years.

6 Conclusion

We have examined the impact of different degrees of fiscal feedback on debt in an economy with nominal inertia where monetary policy is optimal. Consumers are either infinitely lived, or of the Blanchard Yaari type. Our focus is on the extent to which different speeds of fiscal feedback on debt enhance or detract from the ability of the monetary authorities to stabilise output and inflation.

Using a welfare function derived from utility, we find the optimal level of fiscal feedback to be small. With this optimal degree of fiscal feedback, the behaviour of debt is very close to a random walk following shocks. Under a fully optimal fiscal policy, if consumers are infinitely lived we find that debt behaves as an exact random walk, as found in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004). However, if consumers are of the Blanchard Yaari type, the fully optimal fiscal policy is no longer an exact random walk, although it is close to it. We directly infer that optimal monetary policy is active, both for fully optimal fiscal policy and for the optimal level of fiscal feedback. In addition, we find that the costs of restricting fiscal policy to only respond to debt disequilibrium are small compared to a fully optimal fiscal policy.

There is a discontinuity in the behaviour of monetary policy and welfare either side of this optimal level. As the extent of fiscal feedback increases beyond the optimal level, optimal monetary policy becomes less active because fiscal feedback also tends to deflate inflationary shocks. However this fiscal stabilisation is less efficient than monetary policy, and so welfare declines. In contrast, if fiscal feedback falls below the optimal level, either the economy becomes indeterminate, or optimal monetary policy becomes strongly passive, with interest rates falling following a cost-push shock. In effect the focus of monetary policy becomes stabilising debt, because of insufficient fiscal feedback. We get a policy regime with similarities to the Fiscal Theory of the Price Level. We show that this passive monetary policy leads to a sharp deterioration in welfare.

Mortality rate		$p = 0.00$	$p = 0.01$
	Social Loss		
Optimal policy		0.7495	0.7514
	$\lambda = 0.0$	0.9579	0.9577
Simple fiscal	$\lambda = \lambda^*$	0.7500022	0.75210
	$\lambda = \lambda^{\min}$	0.7500009	0.75205
feedback	$\lambda = 0.1$	0.7676	0.7682
	$\lambda = 0.5$	0.8350	0.8353
	Monetary Feedback on Cost-Push Shock (θ_μ)		
Optimal policy		2.1957	2.1257
	$\lambda = 0.0$	-5.3782	-5.3762
Simple fiscal	$\lambda = \lambda^*$	2.2047	2.1316
	$\lambda = \lambda^{\min}$	2.2048	2.1342
feedback	$\lambda = 0.1$	1.6084	1.5871
	$\lambda = 0.5$	0.1714	0.1658

Table 1: Welfare Loss and Activity of Monetary Policy

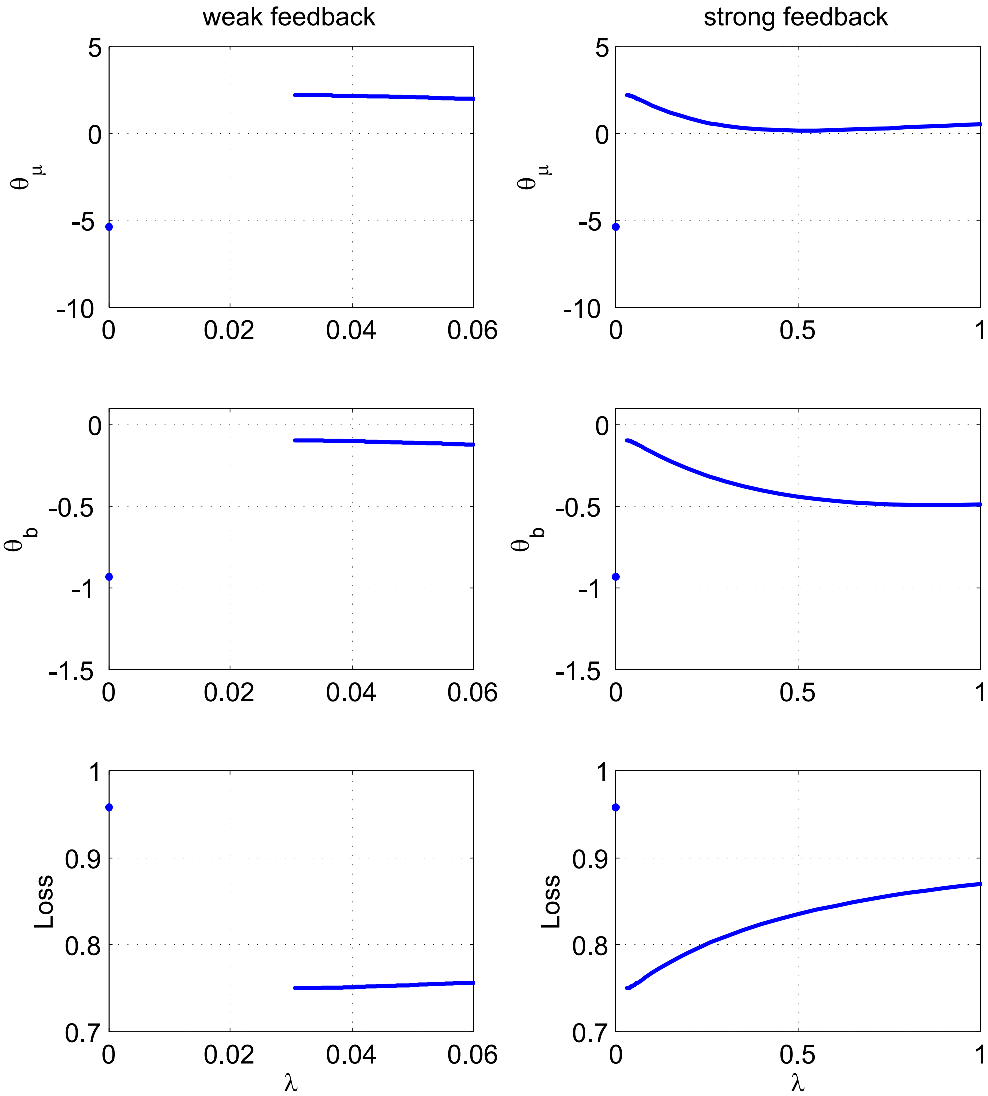

Figure 1: Feedback coefficients of the monetary policy reaction function on the cost push shock and debt, and the resulting value of loss as functions of the fiscal feedback coefficient. The model with infinitely lived consumers.

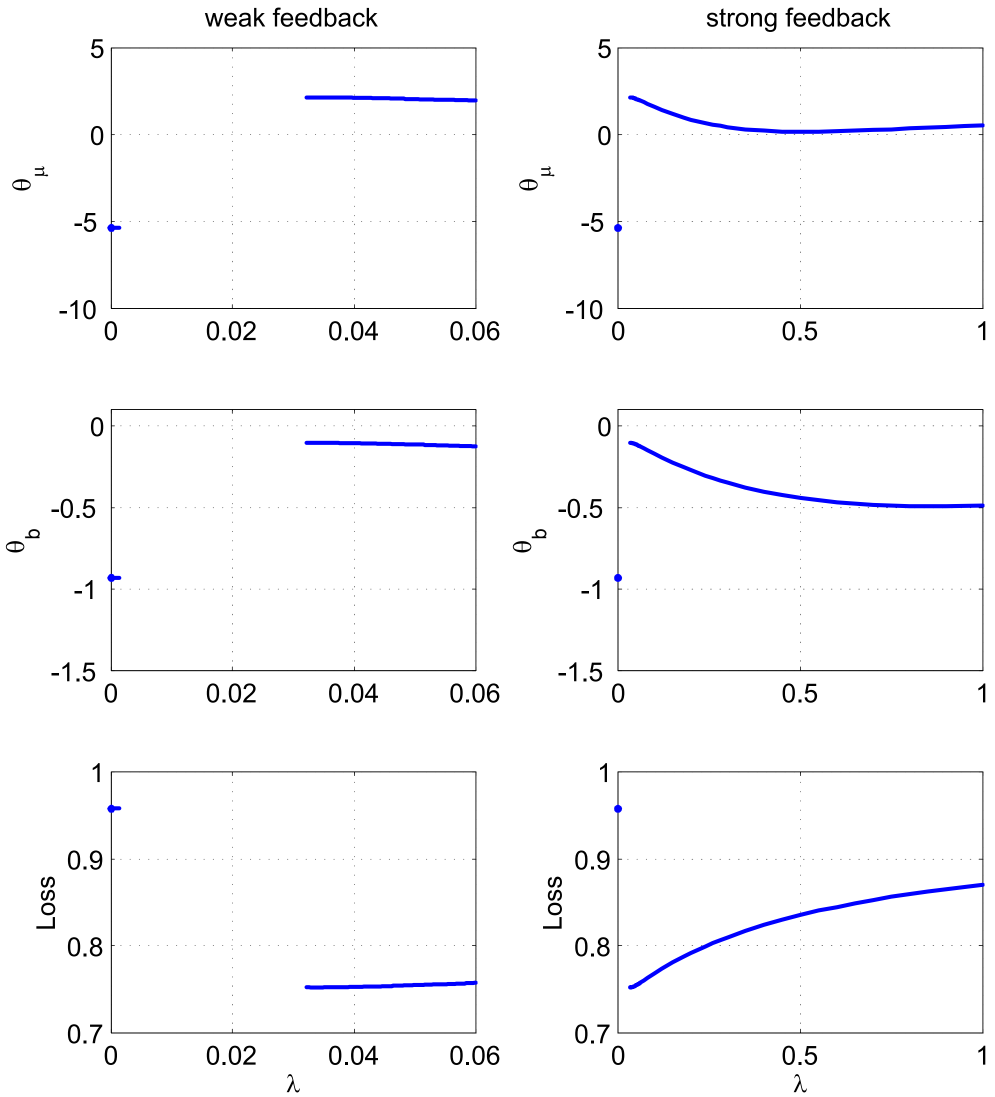

Figure 2: Feedback coefficients of the monetary policy reaction function on the cost push shock and debt, and the resulting value of loss as functions of the fiscal feedback coefficient. The model with Blanchard-Yaari consumers.

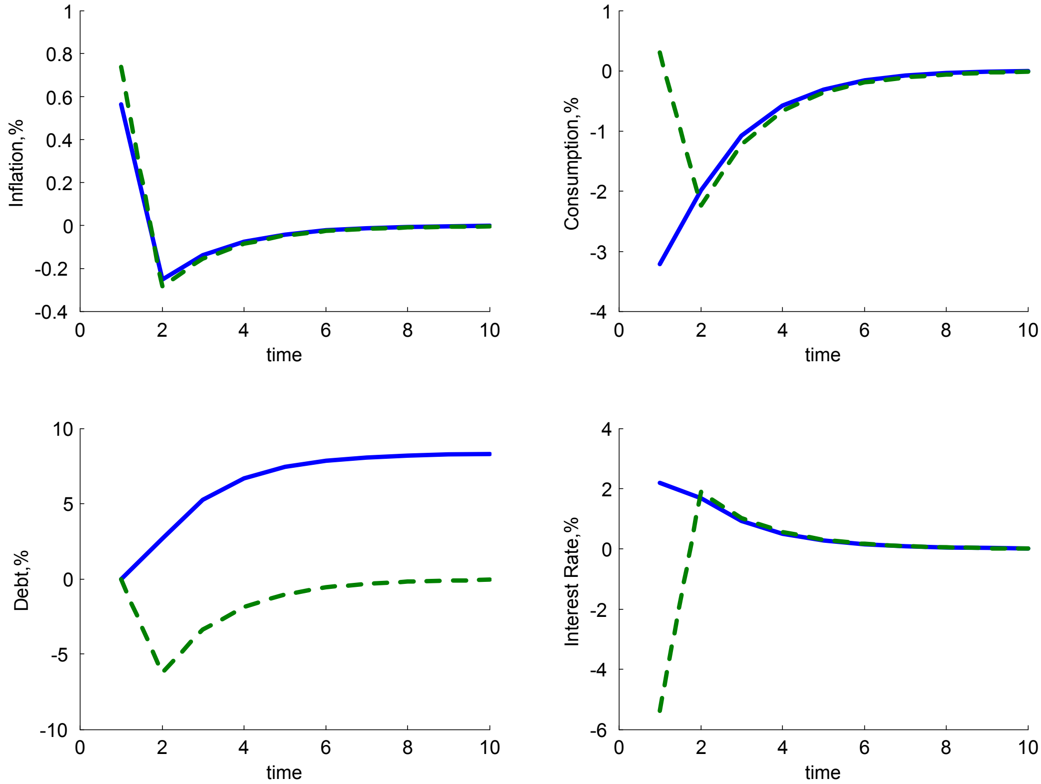

Figure 3: Impulse responses to a cost push shock. The model with infinitely lived consumers. The solid line denotes responses for $\lambda = \lambda^{\min}$ and the dashed line denots responses for $\lambda = 0$.

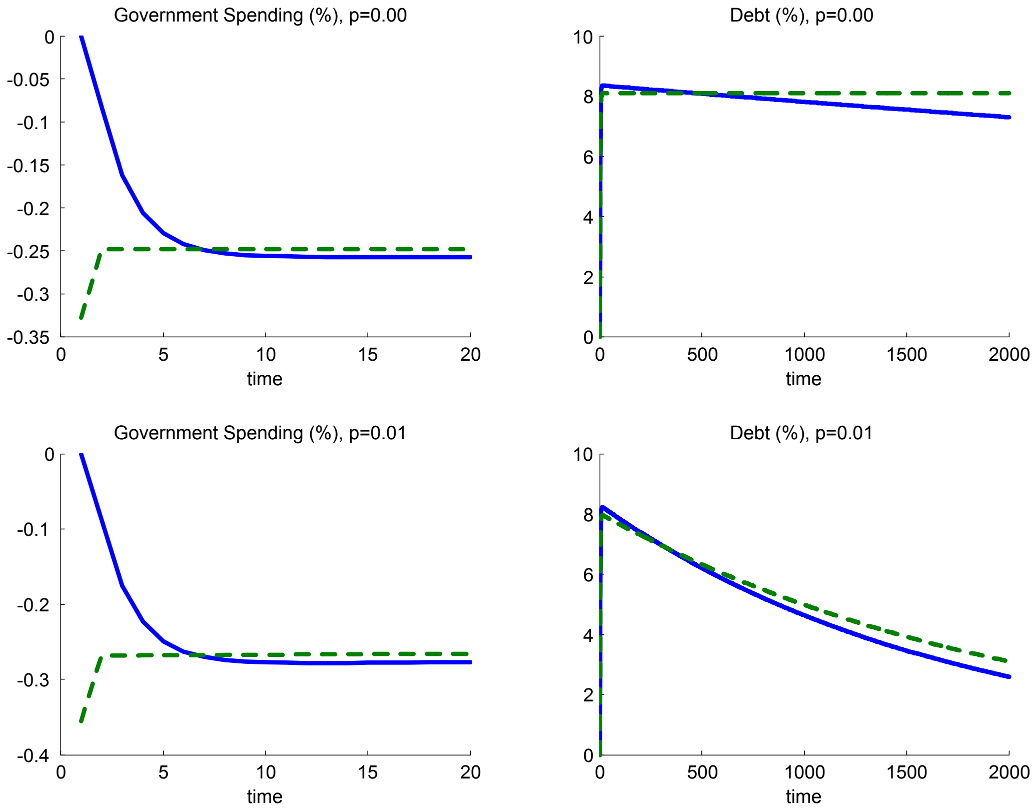

Figure 4: Evolution of Debt and Government spending following a unit cost-push shock. Solid line – optimal simple feedback fiscal policy ($\lambda^{\min}$), dashed line – fully optimal fiscal policy.

A Optimisation, Eigenvalues and Stability

The linear difference system (22)-(28) is closed with a dynamic process for the exogenous shock $\hat{\mu}_t$,and this makes it to be of eighth order. It should be solved subject to eight boundary conditions. We require initial values for predetermined endogenous variables (debt in our case). Principle maximum by Pontryagin requires setting zero initial conditions for those Lagrange multipliers which are associated with dynamic constraints on non-predetermined variables (see e.g. Currie and Levine (1993)). Appropriate transversality conditions close the system.

The system (22)-(28) plus one dynamic process for the shock has eight generalised eigenvalues. It is possible to compute them analytically for two particular values of the fiscal feedback parameter λ.

For $\lambda = 0$ the generalised eigenvalues are (in accending order)

$$\begin{aligned}
\delta_1 &= \delta_2 = 0, \\
\delta_3 &= \frac{1}{2}\left(\frac{\left((\beta+1)\left(a_c+\rho^2 a_y\right)+(\kappa_c+\rho\kappa_y)^2\right)}{\beta\left(a_c+\rho^2 a_y\right)} - \sqrt{\left(\frac{\left((\beta+1)\left(a_c+\rho^2 a_y\right)+(\kappa_c+\rho\kappa_y)^2\right)}{\beta\left(a_c+\rho^2 a_y\right)}\right)^2 - \frac{4}{\beta}}\right) < 1, \\
\delta_4 &= \frac{1}{\beta},\ \delta_5 = 1, \\
\delta_6 &= \frac{1}{2}\left(\frac{\left((\beta+1)\left(a_c+\rho^2 a_y\right)+(\kappa_c+\rho\kappa_y)^2\right)}{\beta\left(a_c+\rho^2 a_y\right)} + \sqrt{\left(\frac{\left((\beta+1)\left(a_c+\rho^2 a_y\right)+(\kappa_c+\rho\kappa_y)^2\right)}{\beta\left(a_c+\rho^2 a_y\right)}\right)^2 - \frac{4}{\beta}}\right) > \frac{1}{\beta}, \\
\delta_7 &= \delta_8 = \infty > \frac{1}{\beta}.
\end{aligned}$$

Similarly, for $\lambda = \lambda^* = \frac{B(1-\beta)(\kappa_c+\rho\kappa_y)}{(1-\rho)(\kappa_c(1-\tau)+\kappa_y\rho)}$ the eigenvalues are

$$\begin{aligned}
\delta_1 &= \delta_2 = 0, \\
\delta_3 &= \text{solution of quadratic equation} < 1, \\
\delta_4 &= 1,\ \delta_5 = \frac{1}{\beta}, \\
\delta_6 &= \text{solution of quadratic equation} > \frac{1}{\beta}, \\
\delta_7 &= \delta_8 = \infty > \frac{1}{\beta}.
\end{aligned}$$

We can also show numerically that for a relatively wide range of $\lambda \geq 0$ the following holds:

$\|\delta_7(\lambda)\| = \|\delta_8(\lambda)\| = \infty$, $\|\delta_1(\lambda)\| = \|\delta_2(\lambda)\| = 0$, $\|\delta_3(\lambda)\| < 1, \|\delta_6(\lambda)\| > \frac{1}{\beta}$.

Eigenvalues $\delta_4(\lambda), \delta_5(\lambda)$ behave in the following way. If $0 \leq \lambda \leq \lambda^*$, $\delta_4(\lambda)$ decreases from $\delta_4(0) = \frac{1}{\beta}$

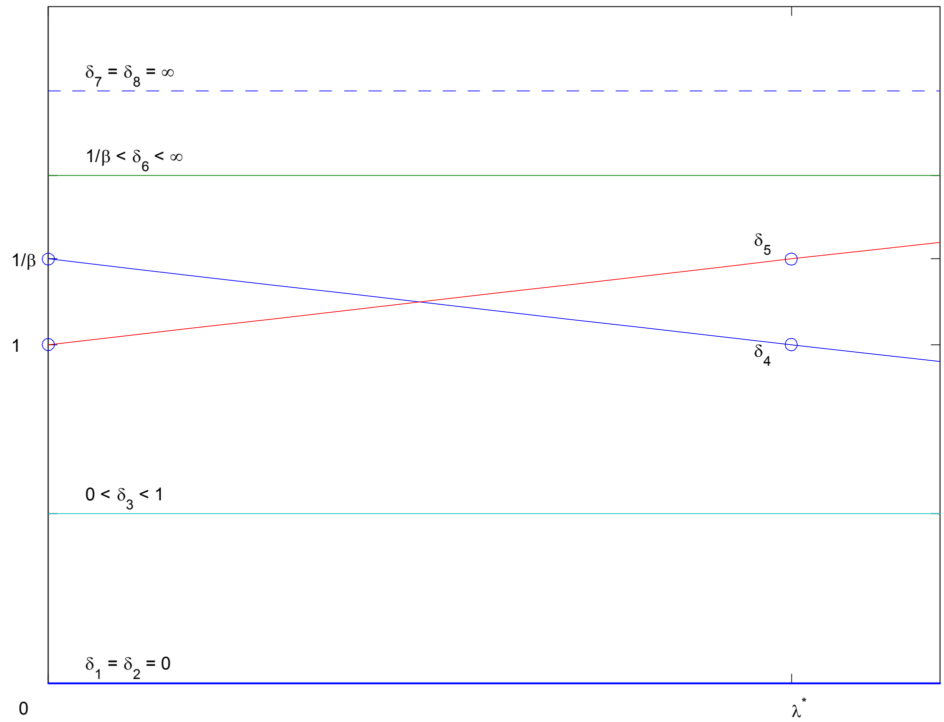

Figure 5: The structure of eigenvalues.

to $\delta_4(\lambda^*) = 1$ and $\delta_5(\lambda)$ increases from $\delta_5(0) = 1$ to $\delta_5(\lambda^*) = \frac{1}{\beta}$. For $\lambda > \lambda^*$, $\delta_4(\lambda) < 1$ and $\delta_5(\lambda) > \frac{1}{\beta}$. This is shown schematically in Figure 5, where eigenvalues are plotted against the value of fiscal feedback λ.

Large fiscal feedback ($\lambda \geq \lambda^*$)

When $\lambda > \lambda^*$ we have four eigenvalues which are strictly less than one, and four explosive eigenvalues (which are strictly greater than $\frac{1}{\beta}$). We thus obtain a unique solution. When t increases, all economic variables, B_t, c_t, π_t, once disturbed, converge to their steady state values.

When $\lambda = \lambda^*$ then $\delta_4 = 1$ and $\delta_5 = \frac{1}{\beta}$. The fifth eigenvalue is classified as explosive (it is ruled out by the transversality conditions.) The solution thus has unit-root dynamics of b_t, g_t and c_t in response to shocks.

Zero feedback ($\lambda = 0$)

Suppose $\lambda = 0$. We now have $\delta_4 = \frac{1}{\beta}$, $\delta_5 = 1$. We take δ_4 as an explosive eigenvalue. Note that in this case neither of the economic variables b_t, c_t, π_t or instrument i_t will exhibit unit-root behaviour.

Lagrange multipliers have unit root dynamics instead.

Positive but small feedback ($0 < \lambda < \lambda^*$)

For the intermediate range of parameter λ, $0 < \lambda < \lambda^*$, there are five eigenvalues that are less than $\frac{1}{\beta}$. We have a continuum of solutions that satisfy transversality conditions and initial conditions. Under any of these continuum of solutions, the model exhibits explosive behaviour. However, this explosive behaviour is modest, as variables will grow at a rate that is slower than the steady state rate of interest ($\frac{1}{\beta}$). The implied loss is finite.

B Optimal solution in a form of feedback rule

It is informative to write down the optimal solution in the form of a feedback rule. System (22)-(28) is a linear system of first-order difference equations which can be written in the following matrix form:

$$\Gamma_0 z_t = \Gamma_1 z_{t-1} \tag{32}$$

where $z_t = \left[\hat{\mu}_t, \hat{\xi}_t, b_t, L_t^{\pi}, L_t^c, i_t, L_t^b, c_t, \pi_t\right]' = [z_{1t}', z_{2t}']'$ is vector of variables, and we have initial conditions for z_{1t}. Matrices Γ_0 and Γ_1 can be singular. Having decided which n_2 eigenvalues are declared as explosive, we can partition our variables correspondingly, where $z_s = (z_{1,s}, z_{2,s})'$. The canonical form (32) can be transformed through a generalised Schur decomposition (QZ) of Γ_0 and Γ_1 (see e.g. Klein (2000)). There exist matrices Q, Z, S and T such that $Q'SZ' = \Gamma_0, Q'TZ' = \Gamma_1, QQ' = ZZ' = I$, and S and T are upper-triangular. The QZ decomposition always exists. We can transform equation (32) into the following form:

$$\begin{aligned} Q'SZ'z_s &= Q'TZ'z_{s-1} \\ (QQ')\, SZ'z_s &= (QQ')\, TZ'z_{s-1} \\ SZ'z_s &= TZ'z_{s-1} \end{aligned}$$

Let $w_t = Z'z_t$ and write the last equation in more detailed bloc-matrix form:

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix}$$

The generalised eigenvalues are defined as $\delta_i = t_{ii}/s_{ii}$. By re-ordering their columns and rows we can achieve any order of generalised eigenvalues along the main diagonals of S and T.

Suppose the partitioning is such that all generalised eigenvalues of the second bloc δ_i should not affect the solution (they are either explosive, $\|\delta_i\| > \frac{1}{\beta}$, or they are ruled out by our selection procedure, for $0 \leq \lambda < \lambda^*$). Then we have to assume that $w_{2,0}$ are zero (and thus $w_{2,t} \equiv 0$, for any t). This leads to the following expression for the remaining variables (S_{11}^{-1} is now invertible as it is upper triangular and does not contain zeros along the main diagonal):

$$w_{1,t} = S_{11}^{-1} T_{11} w_{1,t-1}$$

Now, coming back to the original variables:

$$\begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} = z_t = Z w_t = \begin{bmatrix} Z_{11} \\ Z_{21} \end{bmatrix} w_{1,t} = \begin{bmatrix} Z_{11} S_{11}^{-1} T_{11} \\ Z_{21} S_{11}^{-1} T_{11} \end{bmatrix} w_{1,t-1} = \begin{bmatrix} Z_{11} S_{11}^{-1} T_{11} Z_{11}'^{-1} \\ Z_{21} S_{11}^{-1} T_{11} Z_{11}'^{-1} \end{bmatrix} z_{1,t-1} = \begin{bmatrix} C \\ N \end{bmatrix} z_{1,t-1}$$

Note that z_2 contains the interest rate, so from $z_{2,t} = N z_{1,t-1}$ it follows that we can write the optimal reaction function as the feedback rule (29) in the main text, where L_t^{π}, L_t^{c} are Lagrange multipliers, associated with constraints on inflation and consumption. Lagrange multiplies themselves can be presented as discounted linear combination of *past* values of π and c. Thus, all right hand side variables in (29) are predetermined (see Currie and Levine (1993)). This representation of the optimal policy is useful in judging whether policy is active or passive: we can look at the sign (and size) of θ–coefficients, as they will determine the reaction of the interest rate to shocks in the short run. ϑ–coefficients are set on predetermined Lagrange multipliers, which move relatively slowly in the short run, as they are integrals of past variables.

C Calibration

Given the microfounded nature of the model, there are relatively few parameters to calibrate. One period is taken as equal to one quarter of a year. All behavioural parameters below are taken from Rotemberg and Woodford (1997).

assumed parameters		*assumed parameters*	
$\beta = 0.99$	household discount rate	$\psi = 2.0$	labour elasticity
$\sigma = 0.5$	intertemporal elasticity of substitution	$B/Y = 0.6$	steady state level of debt and assets
$\varepsilon = 5.0$	elasticity of substitution between domestic goods	$\rho = 0.75$	steady state consumption to output ratio
$\Sigma = (0.01)^2 I$	covariance matrix of shocks	$\gamma = 0.75$	probability of that price remains unchanged

As discussed in Appendix D.2, formulae (39) and (37), we assume a subsidy that eliminates the monopolistic and taxation distortions, so $\frac{f_G}{u_C} = \zeta \left(\frac{1-\rho}{\rho}\right)^{-1/\sigma} = 1$. Our calibration $\rho = 0.75$ implies $\zeta = 1/9$.

D Steady State and Welfare

D.1 Government expenditures in steady state

The aggregate demand relationship (12) always holds along the dynamic path of the economy, which can be differentiated with respect to government expenditures in order to yield the following condition:

$$\frac{\partial Y_t}{\partial G_t} = \frac{\partial C_t}{\partial G_t} + 1 \tag{33}$$

Condition (9) also holds along the dynamic path of the economy. Its differentiation yields:

$$\frac{(1-\tau)}{\mu_t} u_{CC}(C_t) \frac{\partial C_t}{\partial G_t} = v_{yy}(Y_t) \frac{\partial Y_t}{\partial G_t} \tag{34}$$

Both conditions (33) and (34) hold in the steady state and can be solved for $\frac{\partial C_t}{\partial G_t}$ and $\frac{\partial Y_t}{\partial G_t}$:

$$\frac{\partial C_t}{\partial G_t} = -\frac{\rho\sigma}{(\psi+\rho\sigma)}, \quad \frac{\partial Y_t}{\partial G_t} = \frac{\psi}{(\psi+\rho\sigma)}$$

We assume that the steady state level of government expenditures is chosen to maximise the utility function (1) (subject to aggregate demand constraint and aggregate supply conditions), so that in the steady state[15]:

$$\frac{\partial}{\partial G}\left(u(C_s) + f(G) - v(Y_s)\right) = u_C(C)\frac{\partial C}{\partial G} + f_G(G) - v_y(Y)\frac{\partial Y}{\partial G} = 0 \tag{35}$$

From (33), (34) and (35) it follows that in equilibrium:

$$\frac{f_G}{u_C} = \frac{v_y}{u_C}\frac{\partial Y}{\partial G} - \frac{\partial C}{\partial G} = \frac{\psi(1-\tau)\mu^w/\mu + \rho\sigma}{(\psi+\rho\sigma)} \tag{36}$$

Note that for iso-elastic utility components

$$u(C_v) = \frac{C_v^{1-1/\sigma}}{1-1/\sigma}, \qquad f(G_v) = \zeta\frac{G_v^{1-1/\sigma}}{1-1/\sigma}$$

in the steady state:

$$\frac{f_G}{u_C} = \zeta\left(\frac{G}{C}\right)^{-1/\sigma} = \zeta\left(\frac{1-\rho}{\rho}\right)^{-1/\sigma} \tag{37}$$

[15] Derivatives of constraints are equal to zero so we did not include them in the final expression.

D.2 Derivation of the Social Welfare Function

The derivation of the welfare metric is standard and for this model it is explained in detail in Kirsanova, Leith, and Wren-Lewis (2006). The one-period (flow) welfare in (20) is $\mathcal{W}_t$:

$$\mathcal{W}_t = u(C_t) + f(G_t) - \int_0^1 v(y_t(z))dz$$

It can be linearised around the steady state

$$\begin{aligned}\mathcal{W}_t &= Cu_C(C)\left(\hat{C}_t + \frac{1}{2}(1-\frac{1}{\sigma})\hat{C}_t^2\right) + Gf_G(G)\left(\hat{G}_t + \frac{1}{2}(1-\frac{1}{\sigma})\hat{G}_t^2\right) \\ &\quad - Yv_y(Y)\left(\hat{Y}_t + \frac{1}{2}(1+\frac{1}{\psi})\hat{Y}_t^2 + \frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})var_z\hat{y}_t(z)\right) \qquad (38)\end{aligned}$$

where we assumed $\sigma = -u_C/u_{CC}C = -f_G/f_{GG}G$, $\psi = -v_y/v_{yy}Y$.

A second-order approximation of aggregate demand (12) can be written as

$$\hat{C} = \frac{1}{\rho}\left(\hat{Y} - (1-\rho)\hat{G} - \rho\frac{1}{2}\hat{C}^2 - \frac{1}{2}(1-\rho)\hat{G}^2 + \frac{1}{2}\hat{Y}^2\right)$$

so we can substitute consumption in (38) and obtain

$$\begin{aligned}\mathcal{W}_s = \rho u_C &\left((1-\frac{v_y}{u_C})\hat{Y}_s - (1-\rho)(1-\frac{f_G}{u_C})\hat{G}_s - \frac{\rho}{2\sigma}\hat{C}^2 - \frac{(1-\rho)}{2}\left(1+\frac{f_G}{u_C}\frac{(1-\sigma)}{\sigma}\right)\hat{G}_s^2\right. \\ &\left. -\frac{1}{2}\left(\frac{v_y}{u_C}\frac{1+\psi}{\psi}-1\right)\hat{Y}_s^2 - \frac{1}{2}\frac{v_y}{u_C}\frac{\psi+\epsilon}{\psi\epsilon}var_z\hat{y}_s(z)\right)\end{aligned}$$

To transform this equation into a more convenient form that does not include linear terms, we proceed as follows (see Beetsma and Jensen (2004b)). We have derived relationship (36) for f_G/u_C in the steady state. If the government removes monopolistic distortions *and* distortions from income taxation in the steady state using a subsidy

$$\mu^w = \frac{\mu}{(1-\tau)}, \qquad (39)$$

then $f_G/u_C = 1$ and so the welfare function does not contain linear terms. The final formula for social welfare is

$$\mathcal{W}_s = -\rho u_C\left(\frac{\rho}{2\sigma}c_s^2 + \frac{(1-\rho)}{2\sigma}g_s^2 + \frac{1}{2\psi}y_s^2 + \frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})var_z\hat{y}_s(z)\right).$$

Woodford (2003) has shown that

$$\sum_{t=0}^{\infty}\beta^t var_z\hat{y}_s(z) = \sum_{t=0}^{\infty}\beta^t\frac{\gamma\epsilon^2}{(1-\gamma\beta)(1-\gamma)}\pi_t^2$$

so, using the conventional notation for gap variables, we get the final formula for the social welfare function:

$$\mathcal{W}_s = -\frac{\rho\,(\epsilon+\psi)\,\gamma\epsilon}{2\psi(1-\gamma\beta)\,(1-\gamma)} u_C \left(\frac{\psi(1-\gamma\beta)\,(1-\gamma)}{(\epsilon+\psi)\,\gamma\epsilon} \left(\frac{\rho}{\sigma} c_t^2 + \frac{(1-\rho)}{\sigma} g_t^2 + \frac{1}{\psi} y_t^2 \right) + \pi_t^2 \right)$$

which is formula (21) in the main text.

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