Can Bernanke Save QE? Bond Supply Shocks in a DSGE Model with a Bernanke, Gertler, and Gilchrist Style Financial Accelerator Mechanism

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Contents

List of Figures

List of Tables

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Abstract

This thesis inserts a household preference over bond maturities into a costly state verification financial accelerator model based on Christiano, Motto, and Rostagno's 2014 AER paper . It is thus building both on the branch of the financial frictions literature that originated with Bernanke, Gertler, and Gilchrist's 1999 paper, and more recent post-crisis attempts to study the macroeconomic affects of quantitative easing programs within the DSGE framework. It uses a sufficiently large 'risk shock' a la Christiano et al. (2014) to introduce a crisis. And then introduces a lagged shock to the outstanding quantity of long term bonds and lagged monetary policy shocks calibrated to match the first round of the Federal Reserve's Large Scale Asset Purchases (LSAPs) and accompanying constraints on the short rate. The results of this simulation show that in the model QE first reduces investment, which then later amplifies the post-crisis stock market boom.

Introduction

It is not difficult to observe that the Federal Reserve's unconventional monetary policy implemented in the wake of the Financial Crisis has had an impact. The package of Large Scale Asset Purchases (QE) and forward guidance the Fed has pursued in the wake of the 2007-08 financial crisis has been followed by a boom in the US stock market. However the theoretical effects of QE inside New Keynesian DSGE models has been disappointingly small. For example Chen, Curdia, and Ferrero (2012) find that the second round of QE purchases had a slightly smaller effect than a surprise 25 basis point cut in the Federal Funds Rate. However most DSGE models such as the one in Chen et al. (2012) focus solely on the macroeconomic impact of QE. Arguably, given the observed behavior of the financial markets in response to QE, if QE were to be assessed inside a DSGE model with an imperfectly functioning financial sector it would enhance the potential theoretical impact of the QE program. This thesis does just that.

Two different areas of the literature that have attracted renewed attention in the wake of the financial crisis are used to construct the model.

The first is the literature on financial frictions. The framework for the model in this thesis is the model in Christiano et al. (2014). In their 2014 AER paper Christiano, Motto, and Rostagno (CMR) build a costly state verification financial accelerator mechanism into a modern DSGE model. This branch of the financial frictions literature originates with Bernanke, Gertler, and Gilchrist (1999). The aim of including a financial friction here is two-fold. First, the CMR model includes a proxy for the value of the stock market. Therefore adding QE to this model allows one to assess theoretically the impact of QE on the stock market. Secondly, QE was employed under conditions that were very obviously characterized by imperfections in the financial markets. Using a financial frictions DSGE model adds realism. It allows for the assessment of both the impact QE had on the financial markets directly and how QE's interaction with the financial system affected the macroeconomy. This opens up a new dimension upon which QE could act.

The second area of the literature addresses the modifications to household preferences necessary to give QE a chance to work in these models. This branch of literature breaks Wallace's Irrelevance Result (1981). The Result implies that the composition of government debt has no real effects, breaking it is a necessary first step to studying QE. One story, Preferred Habitat Theory, is that certain households (like pension funds) want to hold assets whose duration matches the duration of their liabilties whilst other households simply seek to maximize expected return. This view dates back to the work of Modigliani and Sutch (1966) and more recently Vayanos and Vila (2009). It is the basis for Chen et al. (2012), who build a DSGE model to assess QE with two groups of households. One can trade in both long and short bonds, but faces a transaction cost from trading in long bonds. The second group can only trade in long bonds but faces no transaction cost. The second more general story (that encompasess the first) is that of imperfect substitutability between assets. James Tobin was the first to emphasize the necessity of considering a range of assets and interest rates for a nuanced analysis of government debt management and monetary policy.

The Model

2.1 The Bond Market and QE Policy

The model's treatment of the bond market follows Harrison (2012). There are two types of bonds: short, and long. Short bonds sell for a unit price at time t, and return R_t units of currency at time t+1. The model is quarterly, therefore these bonds best approximate 3 month US Treasuries. Long bonds are modelled as consol bonds, that is they are perpetuities that exist for an infinite number of periods (unless the government removes them from the market), pay 1 unit of currency each period, and have a value V_t at time t. For the purpose of tractibility it is assumed that in each period t, as well as making the coupon payment of 1 on each long bond, the government rolls over its debt by purchasing the entire stock of long-term debt (B^c_{t-1}) at the market price V_t and making a new issuance of consol bonds B^c_t which are purchased by households for the market price V_t.

The government's nominal budget constraint is therefore:

$$B_t + V_t B^c_t = R_{t-1}B_{t-1} + (1 + V^L_t)B^c_{t-1} + P_t G_t - T_t \tag{2.1}$$

Where G_t= real government spending, and T_t= nominal lump-sum taxes. By defining $B^L_t \equiv V_t B^c_t$ as the total nominal value of long bonds at time t, one can rewrite the government's nominal budget constraint as:

$$B_t + B^L_t = R_{t-1}B_{t-1} + \frac{(1 + V^L_t)}{V^L_{t-1}} B^L_{t-1} + P_t G_t - T_t \tag{2.2}$$

Further define $R_t^L \equiv \frac{1+V_t^L}{V_{t-1}^L}$ as the gross returns on long bonds. Note the dating: V_t^L is unknown at time t-1, therefore R_t^L (the gross return on a long bond purchased at time t-1) is not known for certainty until time t. Thus the government's consolidated nominal budget constraint finally becomes:

$$B_t + B_t^L = R_{t-1}B_{t-1} + R_t^L B_{t-1}^L + P_t G_t - T_t \tag{2.3}$$

The Quantitative Easing (QE) policy is modeled, as in Chen et al. (2012), as an exogenous i.i.d. shock ($\epsilon_{BL,t}$) to the government's autoregressive supply rule for the market value of detrended long-term bonds:

$$\log\left(\frac{B_t^L}{P_t z_t^*}\frac{1}{b^L}\right) = \rho_{BL}\log\left(\frac{B_{t-1}^L}{P_{t-1}z_{t-1}^*}\frac{1}{b^L}\right) - \epsilon_{BL,t} \tag{2.4}$$

Where $\rho_{BL} \in (0,1)$, and $b^L \equiv \frac{B^L}{Pz^*}$ is the steady-state detrended market value of long bonds. The detrending term z_t^* is defined in the discussion of intermediate goods producers. The rule is specified relative to the steady-state value of long-bonds, therefore in the period of the shock the quantity of long-bonds is $\epsilon_{BL,t}\%$ below its steady-state value. In this model the QE policy is a shock to the composition of the outstanding stock of government bonds, compared with the historical behaviour of this series. The household must hold all available bonds. The central bank's purchase of bonds is modelled here as an exogenous distruction of a portion of the supply of long term bonds. Because QE is specified as a mean zero shock, this model does not incorporate the effects of QE that work via changing agents' expectations. As will be seen later on, the second part of the full unconventional monetary policy "package", that is a constraint on the short rate, is also introduced via mean zero shocks (in this case to the Taylor Rule). In light of this, the results of the simulations using this model should be interpreted as showing the impact of QE excluding its influence on expectations over future monetary policy.

There are two possible ways to extend this model to incorporate the role of expectations over unconventional monetary policy actions. These methods were not pursued in this thesis due to time constraints. The first, specific to the QE policy, is to use a rule based expression for QE. For example the one in Ellison and Tischbirek (2014), where the central bank's purchases respond to deviations of output and inflation from target levels, much like a Taylor type rule for the purchase of bonds. In this case hoseholds can anticipate the QE policy actions of the central bank and form a non-zero expectation over the purchases of long-term bonds by the central bank. The second extension is to use anticipated shocks as is done in Del Negro et al. (2013) and in Christiano et al. (2014) (from here on referred to as 'CMR') when implementing "news shocks". This essentially implements an alternative "crisis" policy regime, whilst avoiding the convergence issues common to New Keynesian models at the Zero Lower Bound (ZLB) in Regime Switching methods such as the Occasionally Binding Constraint algorithm designed by Guerrieri and Iacoviello (2015).

2.2 Households

Except for the addition of the 'liquidity concern' term in households' utility function, households are specified as in the CMR model. All households are identical, they are large in number, and competitive. Each household holds a large number of entrepreneurs and every type of differentiated labour. Households consume, invest to produce raw capital (which is then sold to entrepreneurs), buy and hold long and short term bonds, supply labour, and recieve labour income.

The representative household has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \Big\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di - \frac{\nu}{2} \Big(\frac{B_t}{B_t^L} - 1 \Big)^2 \Big\} \tag{2.5}$$

Where β is the household's discount rate, $\zeta_{c,t}$ is a consumption preference shock, C_t is per capita consumption, and h_{it} is labour supply by labor type i.

The crucial modification here is in the 3rd term (the 'liquidity concern'), which is intended to introduce into the model in a reduced fashion Tobin's idea of imperfect substitutability between different assets. Imperfect substitutability, as defined in Tobin and Brainard (1963), implies that if the rate of return on one asset decreases then the fraction of wealth held in that asset will fall and there will be an increase or at least no change in the fraction of wealth held in all other assets. The supporting argument for QE pursued in this model flips the causality of Tobin's argument. The suggestion here is that the Federal Reserve purchases that removed long term US government bonds from private portfolios reduced the rate of return on that class of bonds. And increase the fraction of wealth held in other asset classes.

The liquidity concern is taken directly from Harrison (2012) and originates in Andrés et al. (2004) who first introduced Tobin's concept of imperfect substitutabilty into the DSGE modellling framework. Because the intention of this paper is to analyze the Federal Reserve's QE programs, a large proportion of which involved the purchase of long term US Treasury Bonds, the liquidity concern term is specified as the ratio of the market value of short to long term US government bonds. The simplified story here is that households require short bond holdings to compensate for the risk of holding long bonds. In reality that risk comes from the loss of liquidity involved in holding long term bonds. In this model it comes from the fact that the price of the long bond next period (V^L_{t+1}) is unknown at the time of purchase and therefore the return on long bonds is uncertain.

The 'liquidity concern' term has a bliss point at $\frac{B_t}{B^L_t} = 1$, where the household suffers no disutility from holding bonds. The liquidity concern parameter ν is calibrated so that the term premium in steady state $R^L - R$ matches the average difference between the yield on a 10 year US Treasury and the the yield on a 3 month US Treasury, over the period from 1985Q1 to 2008Q4. The 10 year rate was chosen as the reference long-term bond rate because, whilst the Federal Reserve's QE purchases were spread over a range of maturities they were concentrated around 10 year bonds (Gagnon et al., 2011).

The non-zero steady state term-premium represents a important difference between the functional form for the liquidity concern used here and the one used in Harrison (2012) which is as follows:

$$-\frac{\tilde{\nu}}{2}\left(\delta^b \frac{B_t}{B_t^L} - 1\right)^2 \tag{2.6}$$

Where δ^b is the steady state ratio of the quantity of long to short bonds. Thus the liquidity preference term is zero in steady state, and therefore in Harrison's model the steady state Euler equations on long and short bonds imply an identical steady state interest rate.

It is possible that the chosen specification in equation (2.5) is less appealing, because it forces the assumption that the bliss point is at the point where households hold equal nominal quantities of long and short bonds. An alternative specification that was originally pursued is as follows:

$$-\frac{1}{2}\left(\frac{B_t}{B_t^L} - \nu\right)^2 \tag{2.7}$$

These specifications are identical in steady state (with the appropriate differences in calibration of ν). But, this alternative specification means small swings in bond supplies lead to unrealistically large variations in rates. As will be shown later, the CMR risk shock can be reasonably well replicated with the chosen liquidity concern. However, when the same exercise was attempted using the alternative specification for the liquidity concern in equation (2.7), the dynamics became highly different, and the path of the long rate violated reasonable bounds.

The household faces the following budget constraint:

$$\begin{aligned} & P_t C_t + B_t + B_t^L + \frac{P_t}{\Upsilon^t \mu_{\Upsilon t}} I_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t \\ & \leq \int_0^1 W_{it} h_{i,t} di + R_{t-1} B_{t-1} + R_t^L B_{t-1}^L + Q_{\bar{K},t}\bar{K}_{t+1} + \Pi_t \end{aligned} \tag{2.8}$$

Where I_t is the quantity of investment goods purchased by the household for a price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon t}}$, $\bar{Q}_{\bar{K},t}$ is the price of capital, W_{it} is the wage rate for labour type i, and Π_t is lump-sum payments. Additionally the household faces the following law of motion for the evolution of raw capital($\bar{K}_t$):

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\Big(\frac{\zeta_{i,t}}{\zeta_{i,t-1}}\Big)\right] I_t \tag{2.9}$$

Where the adjustment cost function S has the following functional form, with $x_t \equiv \zeta_{i,t}\frac{I_t}{I_{t-1}}$:

$$S(x_t) \equiv \frac{1}{2}\Bigg\{ \exp\Big[\sqrt{S''}(x_t - x)\Big] + \exp\Big[-\sqrt{S''}(x_t - x)\Big] - 2\Bigg\} \tag{2.10}$$

Where $S'' \equiv S''(x)$ is a parameter calibrated to match the dynamics of investment.

2.3 Government Policy & Resource Constraint

Monetary Policy Rule

The central bank sets the short rate according to a backward-looking Taylor Rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{Y_t}{Y_{t-1}} - \log\mu^*\right)\right] + \frac{1}{400}\epsilon_{m,t} \tag{2.11}$$

Where μ_z^* is the steady state growth of output.

Government's Tax Rule

Government taxes are set according to the following fiscal feedback rule, based on the rule in Eusepi and Preston (2011).

$$t_t - g_t = \Phi_{tax}\left\{\frac{R_t^L b_{t-1}^L + R_{t-1}b_{t-1}}{R^L b^L + Rb}\frac{\pi\mu_z^*}{\pi_t\mu_{z,t}^*}\right\}^{\phi_{tax}}\exp(\epsilon_t^{tax}) \tag{2.12}$$

Where $t_t \equiv \frac{T_t}{P_t z_t^*}$ is the real and detrended value of lump sum government taxes. And $g_t \equiv \frac{G_t}{z_t^*}$ is exogenous government spending. Here the government's surplus of taxes over spending responds to the ratio of real and detrended government liabilites to its steady-state value. Φ_{tax} = a constant set so that in steady-state the tax rule is an identity. $\phi_T > 0$ is a constant, set to be large enough that the the response of taxes to changes in government liabilities will satisfy the consolidated government budget constraint. ϵ_t^{tax} is a shock to the fiscal feedback rule that is zero in steady state.

The government's budget constraint combined with the tax rule, the exogeneity of government spending, and the AR(1) rule for the supply of long term bonds implies that the quantity of short term bonds is determined without the need for an additional rule.

The Resource Constraint

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t\mu_{\Upsilon,t}} + a(u_t)\Upsilon^{-t}\bar{K}_t + \Theta\frac{1-\gamma_t}{\gamma_t}(N_{t+1} - W^e) + D_t \tag{2.13}$$

Where $a(u_t)\Upsilon^{-t}\bar{K}_t$ is the aggregate capital utilization cost of entrepreneurs, $\Theta\frac{1-\gamma_t}{\gamma_t}(N_{t+1} - W^e)$ are the resources consumed by exiting entrepreneurs, and $D_t \equiv \frac{\mu G_{t-1}(\bar{\omega}_t)R_t^k + Q_{\bar{K},t-1}\bar{K}_t}{P_t}$ are the resources expended on monitoring entrepreneurs.

2.4 Production Markets

Labour Market

The treatment of the labour market is as in a standard DSGE model and is identical to CMR. There are differentiated labor services $h_{it}, i \in [0,1]$. Each type of labour is represented by a monopoly union that sets its wage rate W_{it} whilst facing a Calvo-style friction as follows. Each period a fraction $1-\xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$W_{it} = \left(\mu_{z,t}^{*}\right)^{\iota_\mu} \left(\mu_z^{*}\right)^{1-\iota_\mu} \tilde{\pi}_{wt} W_{i,t-1} \tag{2.14}$$

Where

$$\tilde{\pi}_{wt} \equiv \left(\pi_t^{target}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}, \qquad 0 < \iota_w < 1 \tag{2.15}$$

Labour is aggregated via a Dixit-Stiglitz style aggregator by a competitive and representative labour contractor:

$$l_t = \left[\int_0^1 \left(h_{t,i}\right)^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \qquad 1 \leq \lambda_w \tag{2.16}$$

l_t, the homogeneous labour aggregate, is sold to intermediate goods producers at the nominal wage W_t.

Goods Market

Each intermediate good, $Y_{jt},\ j \in [0,1]$ is produced by a different monopolist according to the following production function:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha}\left(z_t l_{jt}\right)^{(1-\alpha)} - \Phi z_t^{*}, & \text{if } \epsilon_t K_{jt}^{\alpha}\left(z_t l_{jt}\right)^{(1-\alpha)} > \phi z_t^{*} \\ 0, & \text{otherwise} \end{cases} \tag{2.17}$$

Where $0 < \alpha < 1$ and ϵ_t is a technology shock (that is covariance stationary). K_{jt} is the quantity of effective capital monopolist producer j uses, and l_{jt} the quantity of homogenous labour they employ. The fixed cost ϕz_t^* is such that the intermediate monopolistic producer earn zero profits in steady state.

There is Calvo pricing of the intermediate goods. In any period a random fraction of intermediate firms, $1 - \xi_p$ can reoptimize their price P_{jt}. The remaining fraction ξ_p set their price as follows:

$$P_{jt} = \tilde{\pi}_t P_{j,t-1} \tag{2.18}$$

Where inflation indexation is as follows:

$$\tilde{\pi}_t = \left(\pi_t^{target}\right)^{\iota} \left(\pi_{t-1}\right)^{1-\iota} \tag{2.19}$$

$\pi_{t-1} \equiv \frac{P_{t-1}}{P_{t-2}}$ is gross inflation. And π_t^{target} is the central bank's target inflation rate.

The homogenous final good Y_t, is produced by a competative representative firm (with Dixit-Stiglitz technology):

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_{f,t}}} dj\right]^{\lambda_{f,t}}, \; 1 \leq \lambda_{f,t} < \infty, \; j \in \left[0, 1\right] \tag{2.20}$$

The homogenous final good has two uses: consumption and investment. One unit of Y_t can be converted into one unit of the consumption good C_t, and thus (given perfect competition in the use of this technology), consumption has the price P_t. One unit of Y_t can also be converted into $\Upsilon^t \mu_{\Upsilon,t}$ units of the investment good, and thus (again given perfect competition in the use of the technology) has the price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}}$, $\Upsilon > 1$.

There are two sources of growth in the model. First, the trend rise in the aformentioned technology for producing investment goods, Υ^t. And second, the shock z_t which has a stationary growth rate. The detrending term z_t^* is a combination of both sources of growth:

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}. \tag{2.21}$$

z_t^* is used to normalize variables to find a nonstochastic steady state. z_t^* is such that $\frac{Y_t}{z_t^*}$ will converge to a constant in the nonstochastic steady state of the model.

2.5 Entrepreneurs

The characterization of the financial friction is from CMR and is based on the costly state verification (CSV) financial accelerator mechanism in Bernanke et al. (1999), hereafter refered to as 'BGG'. BGG emphasize the following intuition behind the accelerating affects of the financial friction in the CSV class of models. The basic idea is that a fall in entrepreneurial net worth means that the entrepreneurs will have less inside funds to invest in the project. Therefore the mutual funds that make loans to entrepreneurs face a greater agency cost when they finance the entrepreneurs. Essentially the entrepreneur has less "skin in the game". The higher agency cost means mutual funds charge a higher interest rate, so that the premium on external finance faced by entrepreneurs increases. Faced with a larger interest rate on loans, other things being equal, entrepreneurs will choose to purchase less capital. Because, as will be described below, entrepreneurs play a key role in turning raw capital into effective capital used by producers, this rise in the external finance premium will decrease output. The net worth of entrepreneurs is procyclical. Therefore the external finance premium is countercyclical. Thus the interactions between mutual funds and entrepreneurs via the hike in the external finance premium will serve to exacerbate pre-existing cyclical downturns.

Entrepreneurs have the role of turning raw capital (purchased from households) into effective capital (to be then sold to intermediate goods producers). Their aggregate net worth is considered a proxy for the value of the stock market. Entrepreneurs can either be interpreted as being firms in the non-financial sector, or financial institutions with non-diversified holdings.

Entrepreneurs are classified by their net worth, an entrepreneur with net worth $N \geq 0$ is called an 'N-type' entrepreneur. The timing of one cycle in the life of an entrepreneur is as follows. Following production in period t, each entrepreneur gets a loan from a mutual fund. Each mutual fund is specialized. It makes loans only to entrepreneurs of a specific level of net worth, but perfectly diversifies by holding a large number of those loans. The entrepreneur combines the loan $B_{t+1}^{N,credit}$ with their own net worth to purchase raw capital:

$$B_{t+1}^{N,credit} + N = Q_{\bar{K},t}\bar{K}_{t+1}^{N} \tag{2.22}$$

After raw capital is purchased each entrepreneur receives an idiosyncratic shock ω that determines the amount of effective capital they have, $\omega\bar{K}_{t+1}^{N}$. As in BGG, ω is distributed (independently across entrepreneurs and time) log-normally with a unit mean and a standard deviation $\sigma_t \equiv \sqrt{var(\log \omega)}$. σ_t is called the *risk shock*; it can stochastically vary across time.

After receiving the risk shock entrepreneurs must choose the utilization rate of effective capital u_{t+1}^{N}. This determines the amount of effective capital they supply: $u_{t+1}^{N}\omega\bar{K}_{t+1}^{N}$, in return for the market rental rate r_{t+1}^{k}. Entrepreneurs choose the utilization rate to maximize their return on capital: ωR_{t+1}^{k} at the end of period $t+1$. For simplicity all non-lump-sum taxes from CMR have been dropped. The return on capital is defined as follows:

$$R_{t+1}^{k} \equiv \frac{[u_{t+1}r_{t+1}^{k} - a(u_{t+1})]\Upsilon^{-(t+1)}P_{t+1} + (1-\delta)Q_{\bar{K},t+1}}{Q_{\bar{K},t}} \tag{2.23}$$

Where the utilization cost of capital, $a(u_t)$, is increasing and convex:

$$a(u_t) \equiv \frac{r_t^k}{\sigma_a}\left[\exp\Big(\sigma_a(u_t-1)\Big) - 1\right] \tag{2.24}$$

In addition to the utilization choice entrepreneurs must also make a choice over the type of debt contract to accept. It is each entrepreneur's objective to maximize their expected net worth in the next period (t+1), which is as follows:

$$E_t\left\{\int_{\bar{\omega}_{t+1}}^{\infty}\left[R^k_{t+1}\omega Q_{\bar{K},t}\bar{K}^N_{t+1} - B^{N,credit}_{t+1}Z_{t+1}\right]dF(\omega,\sigma_t)\right\}$$
$$= E_t\left[1-\Gamma(\bar{\omega}_{t+1})\right]R^k_{t+1}L_tN \quad (2.25)$$

$\Gamma_t(\bar{\omega}_{t+1}) \equiv \left[1 - F_t(\bar{\omega}_{t+1})\right]\bar{\omega}_{t+1} + G_t(\omega_{t+1})$ is the fraction of expected earnings paid to the mutual fund. Where $1 - F_t(\bar{\omega}_{t+1})$ is the probability the entrepreneur experiences a idiosyncratic shock over the default threshold $\bar{\omega}_{t+1}$, where $F()$ is the cumulative distribution function of ω. And $G_t(\bar{\omega}_{t+1})$, defined as follows, is the expected value of the idiosyncratic shock in the population of defaulting entrepreneurs:

$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) \quad (2.26)$$

$L_t \equiv \frac{Q_{\bar{K},t}\bar{K}^N_{t+1}}{N}$ is the entrepreneur's leverage. Entrepreneurs maximize (2.25) by choosing the conditions that characterize the debt contract, subject to (2.28) described below. The conditions are: one, the level of the idiosyncratic shock ω below which they will default (this is $\bar{\omega}_t$), or equivalently the gross nominal interest rate on debt to be paid next period: Z_{t+1}. And two, the amount of leverage L_t they will take on.

Thus the debt contract is as follows.

$$R^k_{t+1}\bar{\omega}_{t+1}Q_{\bar{K},t}\bar{K}^N_{t+1} = B^{N,credit}_{t+1}Z_{t+1} \quad (2.27)$$

The relevant constraint on the entrepreneurs maximization problem is the mutual fund's cash constraint. That is mutual funds will only sell contracts that yield zero or positive profits. To raise the funds used to make loans to entrepreneurs mutual funds take deposits in an equal quantity ($B^{N,credit}_{t+1}$) from households for which they pay the risk-free short rate R_t. Therefore their cash constraint is as follows:

$$\Big[1 - F_t(\bar{\omega}_{t+1})\Big] Z_{t+1} B_{t+1}^{N,credit} + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R_{t+1}^k Q_{\bar{K},t} \bar{K}_{t+1}^N \geq B_{t+1}^{N,credit} R_t \tag{2.28}$$

The parameter μ is key to the costly state verification (CSV) nature of this specification for the financial sector. μ is the proportional cost that a mutual fund must pay to observe an entrepreneur's return. There is no incentive to pay it when entrepreneurs are able to pay their loan back in full, as indicated by the first term in (2.28). However, if an entrepreneur experiences an idiosyncratic shock ω below the threshold $\bar{\omega}_t$, then they will not be able to repay their debt to the mutual fund and will declare bankruptcy. In this instance the mutual fund only knows that the entrepreneur is bankrupt, but does not observe the value of ω. Without further action by the mutual fund the entrepreneur could decide to transfer only a fraction of their remaining assets, $\omega R_{t+1}^k Q_{\bar{K},t} \bar{K}_{t+1}^N$, back to the mutual fund. In order to become fully informed about the assets a bankrupt entrepreneur has, the mutual fund must pay a cost that is a proportion μ of the final assets recovered. Thus mutual fund only receives a fraction $(1-\mu)$ of the total assets of bankrupt entrepreneurs.

Because of free entry this cash constraint, (2.28) holds with equality. Using the definition of the debt contract above, rearranging, and substituting out $Z_{t+1} B_{t+1}^{N,credit}$ gives the following single constraint that the Entrepreneur's face in maximizing net worth:

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{L_t - 1}{L_t} \frac{R_t}{R_{t+1}^k} \tag{2.29}$$

The resulting FOC is as follows:

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \left[\frac{R_{t+1}^k}{R_t} \Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \Big) - 1 \right] \right\} = 0 \tag{2.30}$$

Aggregates

The aggregates are given in CMR as follows:

Aggregate raw capital:

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN \tag{2.31}$$

Aggregate effective capital:

$$K_t = \int_0^\infty \int_0^\infty u_t^N \omega \bar{K}_t^N f_{t-1}(N) dF(\omega) dN = u_t \bar{K}_t \tag{2.32}$$

Aggregate net worth:

$$N_{t+1} = \int_0^\infty N f_t(N) dN \tag{2.33}$$

Aggregate credit:

$$B_{t+1}^{credit} = \int_0^\infty B_{t+1}^N f_t(N) dN = \int_0^\infty \Big[Q_{\bar{K},t} \bar{K}_{t+1}^N - N \Big] f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1} \tag{2.34}$$

Finally the evolution of aggregate net worth is:

$$N_{t+1} = \gamma_t \Big[1 - \Gamma_{t-1}(\bar{\omega}_t) \Big] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W^e \tag{2.35}$$

2.6 Nested CMR Model

For the purpose of comparison with the original CMR model, a version of that model is used here. It is identical to the above-described model in all respects except for calibration differences in 4 parameters. Crucially the parameter on the liquidity concern, ν, is set to zero. This requires that the discount rate be recalibrated, so that the Euler equations on bonds imply the same short rate in steady state.

Because the rates on both bonds are now identical Φ_T must be recalibrated. And due to the different discount rate ψ_L must be recalibrated, as it is always calibrated to target a quantity of labour hours h equal to unity in steady state. The calibration differences are summarized in the table below:

Table 2.1: Parameter Calibrations Specific to Nested CMR

Parameter	**Description**	**Calibration**
β	Discount Rate	0.9975
ν	parameter on the liquidity concern	0
Φ_T	steady state structural surplus	0.0098
ψ_L	disutility weight on labor	1.1839

For notational simplicity this nested model will be refered to as 'NCMR' from here on. The main model described here will either be refered to as the 'main' model, the 'Liquidity Preference' model or 'LP' model.

Simulation Setup

3.1 Data and Calibration

3.1.1 Data

The period for which this model is calibrated is from 1985Q1 to 2008Q4.

The data on bond supplies come from the Haver/DLX USECON database. The data are on the nominal value of privately-held marketable interest-bearing US public debt broken down by time until maturity. The 'long' bonds are considered bonds with over 1 year until maturity, and the 'short' bonds are bonds with less than 1 year left until maturity. Haver/DLX datacodes are in the table below. The data are monthly. Thus a quarterly dataseries is constructed by taking the end of period observation. Over a year the observations on January, April, July, and October are used to match the St. Louis Federal Reserve's Economic Database (FRED) quarterly GDP data series.

Table 3.1: Haver/DLX USECON Data Codes

Haver/DLX Code	**Data Series**
PDIMP	Total
PDIMPL	Less than 1 year left until maturity
PDIMP1	1 to 5 years left until maturity
PDIMP5	5 to 10 years left until maturity
PDIMP10	10 to 20 years left until maturity
PDIMP20	over 20 years left until maturity

Data on output comes from the FRED database (FRED code: GDP). A dataseries of the ratio of bond quantities to output is constructed as follows. For the short bond:

$$\frac{B_t}{Y_t} = \frac{b_t P_t z_t^*}{y_{z,t} P_t z_t^*} = \frac{b_t}{y_{z,t}} = \frac{PDIMPL}{GDP} \tag{3.1}$$

And for the long bond:

$$\frac{B_t^L}{Y_t} = \frac{b_t^L P_t z_t^*}{y_{z,t} P_t z_t^*} = \frac{b_t^L}{y_{z,t}} = \frac{PDIMP1 + PDIMP5 + PDIMP10 + PDIMP20}{GDP} \tag{3.2}$$

The averages over the 1985 Q1 to 2008 Q4 period of each constructed series are used as the steady state target of each bond type's quantity to output ratio.

Data from the Federal Reserve Board's H.15 Selected Interest Rate Series are used to calibrate the steady state interest rates. The rate on the short bond is matched to the period average of the market yield on U.S. Treasury securities at 3-month constant maturity. The rate on the long bond is matched to the period average of the market yield on U.S. Treasury securities at 10-year constant maturity.

The steady state return on capital is calibrated to match the period average on the US Bank Prime Loan Rate (FRED code: MPRIME). This dataseries is also part of the H.15 Selected Interest Rate Series.

3.1.2 Calibration

Certain parameters, in particular the parameter in the liquidity concern, ν, do not have well-established calibration methods. They are set to match observed averages in the 1985 Q1 to 2008 Q4 US data. Parameters that are calibrated to target specific values in steady state are given in the table 3.2 and described below it.

Table 3.2: Parameters Calibrated to Match a Specific Steady State

Parameter	**Description**	**Calibration**
β	discount rate	0.9948
ν	parameter on the liquidity concern	0.0093
Φ_T	steady-state structural surplus	0.0201
ψ_L	disutility weight on labor	1.1928

Following CMR ψ_L is set so that h (hours worked) is equal to 1 in the steady state.

β and ν are set using equations 2 and 3 in steady state (see appendix A.1 for equation numbering) so that: a) $R^L - R = 0.0042$ (the observed term premium over the period), and b) $R = 1.0117$ (the short rate matches the observation on 3 month US Treasury Bills).

Unless otherwise indicated the following parameters are fixed according to the calibration in CMR, or CMR posterior modes.

Table 3.3: CMR Based Calibrations - Economic Parameters

Parameter	**Description**	**Calibration**
α	capital's share of output	0.4
δ	depreciation rate of capital	0.025
ϵ	steady-state value of the technology shock	1
η_g	steady-state ratio of government spending to output	0.2
λ_f	steady-state markup in the product market	1.2
λ_w	steady-state markup in the labour market	1.05
μ_z^*	mean growth rate of the unit root technology shock	1.0041
σ_L	Frisch elasticity of labor supply	1
Υ	quarterly rate of investment-specific technological change	1.0042
w^e	lump sum transfer from household to the entrepreneur	0.005

Table 3.4: Calibrations from CMR Posterior Modes - Economic Parameters

Parameter	**Description**	**Calibration**
b	habit parameter	0.74
ι	price indexing weight on inflation target	0.9
ι_μ	wage indexing weight on persistent technology growth	0.94
ι_w	wage indexing weight on inflation target	0.49
Θ	determines the resources used for state-verification in resource constraint	0.005
μ_Υ	steady-state value of $\mu_{\Upsilon,t}$	1
μ	monitoring Cost	0.21
ϕ_π	parameter on inflation in the Taylor Rule	2.3965
ϕ_y	parameter on output in the Taylor Rule	0.3649
ϕ_T	parameter in the fiscal feedback rule (Calibrated from Chen et al. (2012))	1.3147
π^{target}	steady-state target inflation (calibrated to target a 2% annual inflation rate)	1.005
ρ_m	weighting of lagged short-rate in Taylor Rule	0.85
S''	parameter in the investment adjustment cost function, calibrated to match model specific dynamics of investment	20
σ_a	curvature of utilization cost	2.54
ξ_p	Calvo price stickiness	0.74
ξ_w	Calvo wage stickiness	0.81
ζ_c	steady-state value of $\zeta_{c,t}$	1
ζ_i	steady-state value of $\zeta_{i,t}$	1

Table 3.5: Shock Autocorrelation Calibrations

Parameter	**Description**	**Calibration**
ρ_{BL}	evolution of long bonds supply, calibrated to match LSAP duration	0.9643
ρ_{ϵ}	transitory technology	0.81
ρ_{g}	government spending	0.94
ρ_{γ}	equity	0
ρ_{λ_f}	price markup	0.91
ρ_{μ_Υ}	investment goods price	0.99
$\rho_{\mu_z^*}$	persistent technology growth	0.15
$\rho_{\pi^{target}}$	inflation target	0.9750
ρ_{σ}	risk shock	0.97
ρ_{ζ_c}	consumption preference shock	0.9
ρ_{ζ_i}	marginal efficiency of investment	0.91

3.2 The Model's Response to a Risk Shock

The main model's response to a risk shock is described by the following stylized facts. The relevant IRFs can be found in appendicies B.1 and B.2. Consumption, output, labour hours, investment, capital, net worth, the price of capital, the utilization rate of capital, and the quantity of aggregate credit should all fall below their steady-state values. The risk shock σ_t, the default threshold $\bar{\omega}_t$, and leverage, should all increase above their steady-state value.

The story behind these stylized facts, told in CMR, is as follows. A risk shock, an increase in the value of σ_t, means that the standard deviation of the distribution of idiosyncratic shocks experienced by entrepreneurs has increased. Therefore the probability of an entrepreneur experiencing a low (bankruptcy-inducing) level of ω increases. Thus a greater proportion of the population of entrepreneurs defaults. The mutual funds are forced, via their cash constraint (2.28), to raise the rate charged on loans to entrepreneurs: Z_{t+1}.

Faced with less favourable borrowing conditions entrepreneurs reduce their demand for credit. So aggregate credit B_{t+1} falls. As a result entrepreneurs purchase less raw capital from households. The fall in demand for raw capital reduces the price of capital, and with it the households' incentives to invest in capital. Investment falls, and thus consumption and output must also fall.

Additionally there is a financial accelerator effect, through the following circular interactions. The fall in the price of capital also reduces entrepreneurial net worth, therefore reducing the inside funds available to purchase capital, and so their demand for capital falls even more. This leads to a further fall in price. Eventually this cycle stabilizes when the price of capital has fallen sufficiently. Inflation drops because of the drop in output. Credit falls by less than net worth, so leverage rises.

One anomaly between the results here and those in CMR is the response of the return on capital. Whilst the shape of the IRF is the same—the return on capital falls on impact of the risk shock and then rises—the magnitude of the fall on impact is greater here. This can be fixed by reducing the calibrated value of the S'' parameter in the investment adjustment cost function, but at the cost of changing the dynamics of most other variables. Therefore S'' is calibrated here to match the dynamics of investment in the nested CMR model with the original CMR model. By doing so the dynamics on all other variables except the return on capital R_t^k are plausible, though they do exhibit an inital spike on impact before declining as expected. It is suspected that this is due to a calibration issue elsewhere in the model, to be fixed in further work when time constraints are less binding, and does not detract from the overall use of the model.

3.3 Mapping Between Actual Purchases and Model Purchases of Long-Term Bonds

As discussed before the model treats quantitative easing as the exogenous distruction of portions of the quantity of long bonds held by households. The size and duration of the QE policy is jointly determined by the following parameters: σ_{BL} and ρ_{BL}. Recall the QE rule is as follows:

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{BL}\log\left(\frac{b_{t-1}^L}{b^L}\right) - \epsilon_{BL,t} \tag{3.3}$$

Where b_t^L is the real detrended quantity of long bonds in period t. The shock standard deviation parameter: σ_{BL} determines the amount of long bonds that are removed from the households' hands in period 0 (the first period of the QE policy), as a percentage of the steady-state quantity of long bonds. Thus:

$$\log\left(\frac{b_0^L}{b^L}\right) = -\sigma_{BL} \tag{3.4}$$

Recall that the long bonds in this model are treated as consol bonds that are rolled over every period by the government. Before the QE shock hits, the quantity of long bonds is equal to the steady-state quantity of long bonds every period. After the QE shock hits, the parameter ρ_{BL} determines the persistence of bond purchases in the following periods. In the extreme case, if ρ_{BL} were equal to zero, then it would be as if the Federal Reserve immediately reversed its QE policy by selling its long bond holdings back to the private sector one period after they were purchased. In reality this is not the case, as the Federal Reserve continues to hold a large number of bonds on its balance sheet. This indicates that this autocorrelation parameter should be calibrated close to unity.

The calibration of ρ_{BL} is as follows. First note that t periods after the initial QE shock the quantity of long bonds "purchased" (i.e. exogenously destroyed) is:

$$\log\left(\frac{b_t^L}{b^L}\right) = -\sigma_{BL}\rho_{BL}^t \tag{3.5}$$

Therefore the cumulative percentage devation from the steady-state supply of bonds is:

$$-\sigma_{BL}\sum_{t=0}^{\infty}\rho_{BL}^t = \frac{-\sigma_{BL}}{1-\rho_{BL}} \tag{3.6}$$

Gagnon et al. (2011) notes that the first round of QE was equal to about 20% of the outstanding stock of 10 year equivalents. Therefore the relationship between the two parameters in question is:

$$\frac{-\sigma_{BL}}{1-\rho_{BL}} = (-0.2)(x) = -1.2 \tag{3.7}$$

Where x= the number of quarters during which QE kept the supply of long bonds below its steady-state level. Thus x would be the number of periods before QE is reversed and the supply of long bonds is allowed to return to its steady-state level. This reversal of QE has not occured yet, so pinning down x and thus ρ_{BL} from the data is not possible. For the purpose of calibrating ρ_{BL} set σ_{BL} to 20%. Thus

$\rho_{BL} = 1 - \frac{1}{x}$. Considering that the first round of QE began in the second quarter of 2009, and has yet to be undone 7 years later at the date of writing, the low end for x is 28. This implies a conservative calibration of $\rho_{BL} = 0.9643$.

In a way this calibration is the most adverse to finding an impact of QE because it underestimates the persistence of QE. An improvement on this specification that can be pursued in future work is to drop the autocorrelation rule entirely and simply specify the QE rule as:

$$\log\left(\frac{b_t^L}{b^L}\right) = -\epsilon_{BL,t} \tag{3.8}$$

Then the path of QE purchases can be entirely introduced via a series of shocks (anticipated and unanticipated) to $\epsilon_{BL,t}$. It is important to note that this specification (3.8) is only prefereable when an anticipated component to the QE shock is included. The main model of this thesis only deals with unanticipated shocks; therefore the QE rule used (3.3) is better because given a non-zero autocorrelation parameter households have a non-zero expectation over QE purchases in the periods after the initial QE shock is observed.

3.4 Financial Crisis Timeline

In order to justify the timing and calibration of the following simulations a brief timeline of the relevant events of the Financial Crisis is included here. The timeline is comprised of selected events from the financial crisis timeline assembled by The Federal Reserve Bank of St. Louis (2016)

Table 3.6: Financial Crisis Timeline

Date	**Event**
September 15, 2008	Lehman Brothers files for bankruptcy
September 16, 2008	FOMC maintains a 2% target for the Federal Funds Rate (FFR)
October 8, 2008	FOMC reduces the FFR target to 1.5%
October 29, 2008	FOMC reduces the FFR target to 1%
December 16, 2008	FOMC reduces the FFR target to a 0 to 0.25% range. Additionally the FOMC press release states that it is "...evaluating the potential benefits of purchasing longer-term Treasury securities."
January 28, 2009	FOMC press release indicates it is ready to purchase longer-term Treasury Securities if the circumstances indicate they will be effective, particularly in improving private credit market conditions.
March 18, 2009	FOMC maintains the 0 to 0.25% target range for the FFR. Additionally it announces purchases of up to $300 billion of longer-term Treasury securities over the next six months.

3.5 Simulation Methods

All simulations are done in Dynare. The crisis simulations, because they start at a vector for endogenous variables that is not equal to the steady state, as will be described below, are done using the dynare function `simult_` (Dynare Team, 2005). This function uses a perturbation approach to simulate the model given a matrix of shock values and a vector of initial values for the endogenous variables. Therefore, as was done here, it is possible to write a matlab script that loops over runs of the dynare .mod file that includes the `simult_` function, to calibrate the necessary path of shock values to target a specific path for the short rate R_t.

Simulations

4.1 The Small Risk Shock

A 10% risk shock is reproduced using the main (liquidity preference) model and the nested CMR model, to show that the response to a small risk shock are similar to the results in the CMR paper. The impulse response functions from these simulations can be found in appendices B.2 and B.1 respectively.

4.2 The Crisis

The "crisis" simulation is characterized as follows. The risk shock, σ_t is calibrated to target a path for the short rate, R_t, that violates the zero lower bound (ZLB) for 6 periods. The zero lower bound is defined as a value of the short rate between 0 and 6.25 basis points (quarterly), which matches the 0 to 25 annual basis points target for the Federal Funds Rate (FFR) during the crisis. The crisis simulation, unlike the above replications, is not initialized from the steady state. Instead all endogenous variables are at their steady-state value, except for R_t, which is set to $R_t = 1.005$ to match a 2% annualized short rate. This is done to match the timeline of the 2007-08 financial crisis, described in table 3.6. Prior to the collapse of Lehman Brothers, which is considered the "crisis" shock here, economic conditions had already induced the Federal Reserve to cut the FFR to 2%.

Additionally the Taylor rule is modified to reflect the changed nature of monetary policy making during "crisis times". The backward looking element of the rule is dropped, so the new "crisis times" Taylor Rule is as follows:

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{Y_t}{Y_{t-1}} - \log\mu_z^*\right) + \frac{1}{400}\epsilon_{m,t} \tag{4.1}$$

Where ϕ_π and ϕ_y are calibrated as before.

This is done to remove the unrealistic inertia to policy making during the "crisis" period. The rationale here is that following the Crisis Shock monetary policy makers care more about current economic conditions than a smooth path for the policy rate. Even with this inertia removed, without any additional help the short rate will still take multiple periods to fall to the ZLB following the Crisis Shock. In reality, see table 3.6, it took the Federal Reserve only one period following the Lehman collapse to go to the ZLB. This is fixed in the model by calibrating expansionary monetary policy shocks so that the short rate attains the ZLB only one period after the risk shock hits. Contractionary monetary policy shocks are then used to keep the short rate from falling below the ZLB, as in some periods the Taylor Rule implied net short rate is negative.

4.3 The Unconventional Monetary Policy Response

Quantitative Easing and a constraint on the short rate are introduced together as a unconventional monetary policy package in this model. In all QE simulations the short rate is constrained to the ZLB for 6 periods beyond the point it ceases to bind in the crisis without unconventional policy intervention.

When introducing QE it is necessary to constrain the short rate to the ZLB , otherwise the stimulatory effects of QE will induce a rise in the short rate via the Taylor Rule. This rise in the short rate would be counterfactual, as the Federal

Reserve maintained the ZLB long into the life of the QE program. In essence, as is emphasized in Chen et al. (2012), it is essential to consider quantitative easing together with a constrained path for the short rate as comprising a complementary "package" of unconventional monetary policy.

It is important to note, as was mentioned before, that the model's treatment of the unconventional monetary policy package does not account for the role of Forward Guidance. In reality the role of Federal Reserve announcements on their future policy actions for the FFR undoubtedly comprised an important part of the policy package. Because the constrained path for the short rate is implemented via unanticipated monetary policy shocks this model does not give the Federal Reserve the ability to affect agents expectations over the future path of the short rate. As mentioned in section 2.1, introducing an anticipated component of shocks to exogenous variables is a technically feasible way of extending this model to incorporate the expectational aspects of unconventional monetary policy.

Due to the nature of the QE specification in this model and the construction of long bonds as consols, the path of asset purchases here cannot directly match the path of asset purchases pursued by the Federal Reserve in reality. According to Gagnon et al. (2011) the first round of Large Scale Asset Purchases entailed purchasing about 20% of the outstanding stock of 10 year equivalents. These purchases were spread out over a number of periods and held for a number of periods after that. For example the simulation Chen et al. (2012) assumes that the Federal Reserve holds its balance sheet constant for two years after the completion of purchases.

However in this model, as can be seen in the following IRFs, the quantity of bonds "exogenously destroyed" (i.e. held by the Federal Reserve and not the public), begins to decay immediately in the periods following the QE shock. There are two points of mismatch between the model and reality. The first, as discussed, the immediate decay of the Federal Reserve's balance sheet. The second mismatch is that the initial QE shock frontloads the purchases. A natual extension to this model, once an anticipated component to shocks is implemented, would be to introduce a sequence of QE shocks so that the QE purchases are spread over time in a more realistic manner.

The baseline QE program considered here is calibrated to match a negative 25% initial shock to the quantity of long bonds. Because of the frontloading and decaying issues this can be considered an approximate but not exact simulation of the first round of QE purchases pursued by the Federal Reserve starting in March 2009. One further departure from the reality of the program is that the model simulation of QE introduces the program in the period immediately following the Crisis Shock (i.e. after 3 months); whereas in reality the Federal Reserve's QE program commenced 7 months after the Lehman Brothers bankruptcy.

4.4 Impulse Response Functions and Discussion

The direction of specific variables reaction to QE will be highlighted and then discussed. With QE (compared to the crisis without QE intervention) consumption is higher and investment is lower. However output is unchanged. Net worth is higher (but this effect is not immediate). Inflation is greater, the long rate is lower, and capital is slightly lower, the price of capital is higher (again not immediately). Credit, leverage, and $\bar{\omega}_t$ are lower (this effect increases in magnitude at the same time as the constraint on the short rate is lifted). With the exception of the price of capital, all of these effects increase in magnitude when the size of the QE policy increases.

QE Mechanism:

The QE mechanism works in two stages. First, it amplifies the stimulus resulting from conventional monetary policy. Second, the lagged impact of the immediate effects serve to amplify the stock market boom that occurs two years following the Crisis Shock.

Immediate Effects: In this model exogenous monetary policy shocks are used to constrain the short rate to the ZLB following the Crisis Shock. The direct impact of QE is to change the relative supply of long and short term bonds in the households' portfolio. Because of the inclusion of the 'liquidity concern' in the households' utility function, the term premium on long bonds must fall because QE has made them more scarce. In the absence of a constraint on the short rate, this adjustment would happen both via a fall in the long rate and an increase in the short rate. Instead, here QE serves to make an identical path for the short rate less contractionary and more expansionary in terms of monetary policy because, given a larger QE program, the term premium is smaller and so the monetary policy shock must force the short rate down by more.

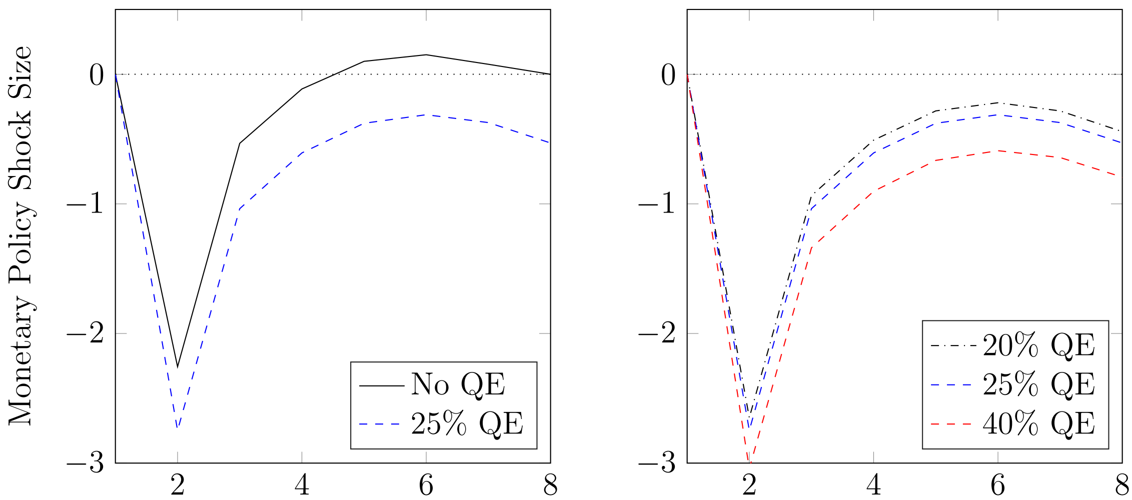

Figure 4.1: Effect of QE on the Path of Monetary Policy Shocks

The lower path of monetary policy shocks indicates that the same path for the short rate translates into more stimulatory monetary policy under QE. In essence QE is turning what is at some points a contractionary monetary policy into a more expansionary monetary policy.

This allows the path of inflation under QE to be above the path of inflation in the absence of unconventional policy intervention. Note, both paths of inflation are below the steady state 2% annualized value of inflation. Therefore, the QE program's implied path for inflation is not counterfactual to the observed low values for inflation in the wake of the 2007-08 financial crisis. The impact on inflation increases with the size of the QE policy:

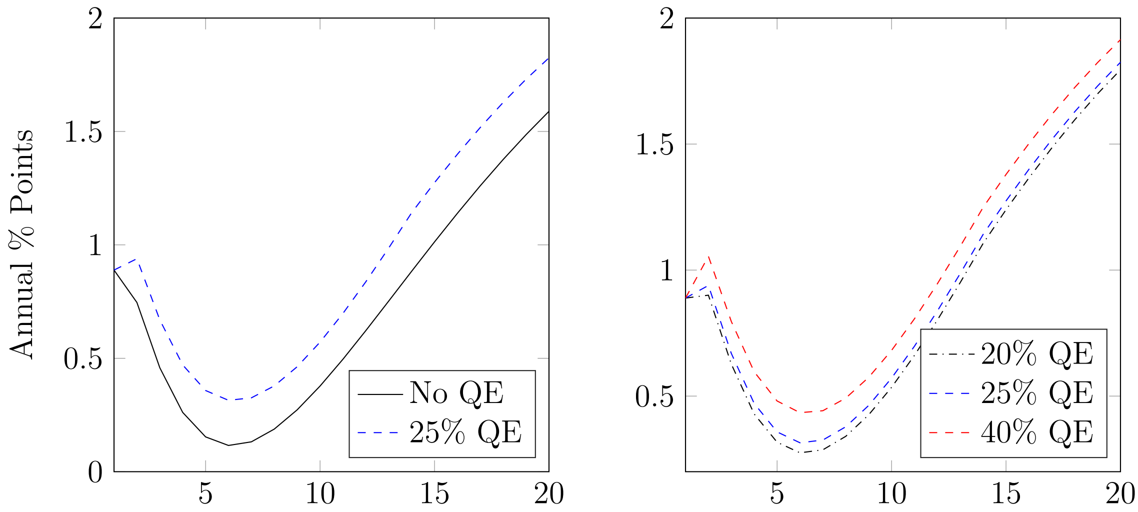

Figure 4.2: QE's Effects on Inflation

The increase in inflation also serves to erode the real value of the short rate. The lower real rate induces households, who are responsible for both consumption and investment decisions, to allocate more funds to consumption and away from investment. This effect is larger in magnitude when the size of the QE policy increases:

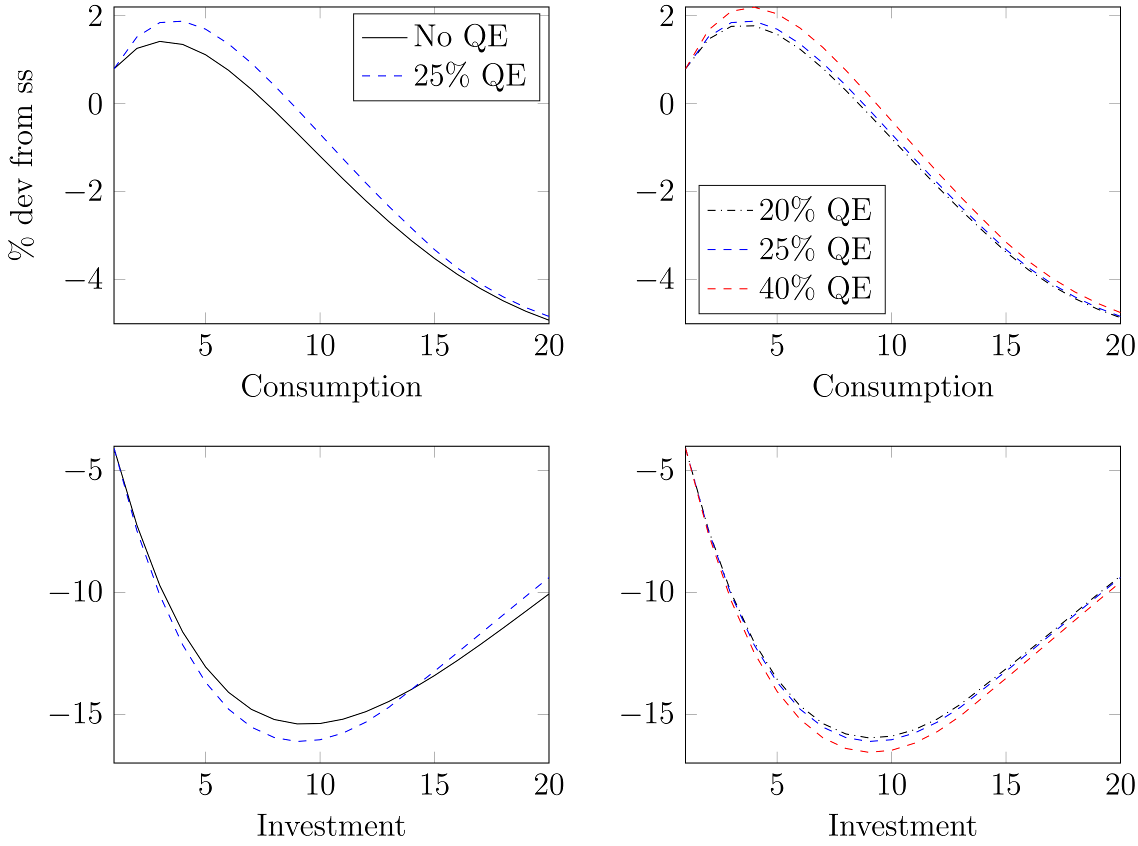

Figure 4.3: QE's Effects on Consumption and Investment

Delayed Effects: As can be seen in figure 4.3 the path of investment under QE eventually crosses and goes above the no-QE investment path. This helps to explain the fact that QE seems to exacerbate the boom in net worth (the proxy in this model for the stock market) two years after the Crisis Shock, as seen in figure 4.4 below:

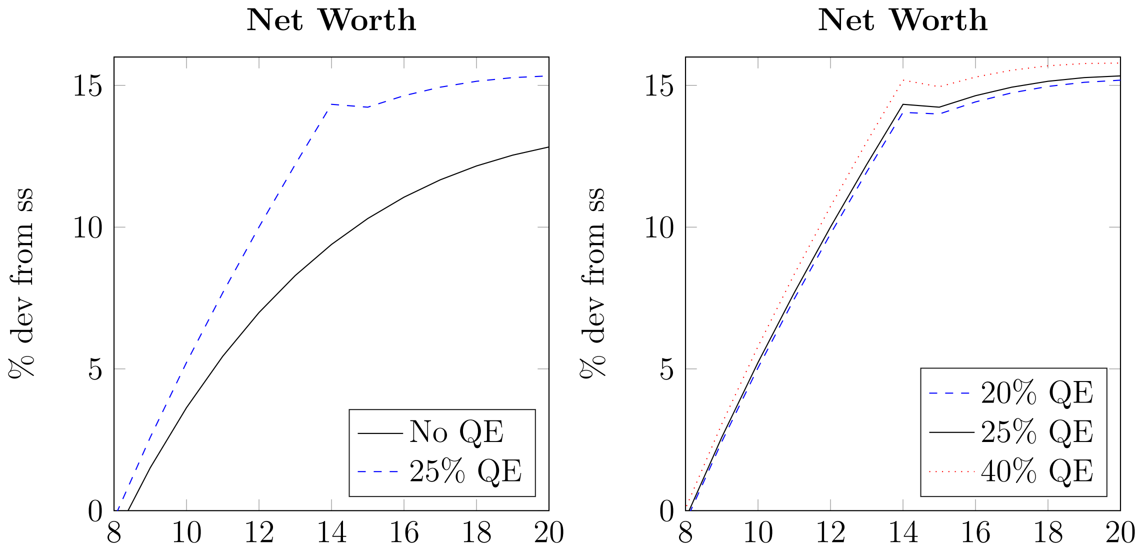

Figure 4.4: Net Worth Two Years After the Crisis

Equation (2.35) gives aggregate net worth:

$$N_{t+1} = \gamma_t \Big[1 - \Gamma_{t-1}(\bar{\omega}_t)\Big] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W^e$$

Because γ_t is exogenous and W^e is constant, both are unaffected by the introduction of QE. Therefore, the stimulatory impact of QE on net worth must come either via the default threshold ($\bar{\omega}_t$), the return on capital (R_t^k), the price of capital ($Q_{\bar{K},t-1}$), or the aggregate quantity of capital ($\bar{K}_t$). Here QE has a negative impact on the aggregate quantity of capital (see fig 4.5 below), which in the absence of other effects would decrease net worth. However QE also reduces the default threshold whilst increasing the return on capital and the price of capital, all effects that positively impact net worth.

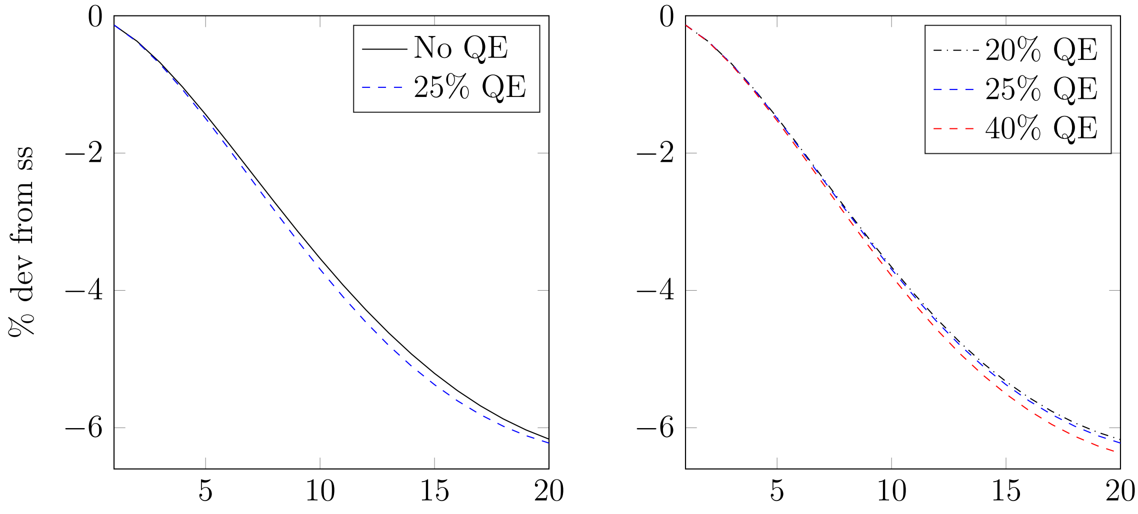

Figure 4.5: Quantity of Capital

The connection between the immediate and delayed effects of QE is the path of investment. QE depresses investment, and thus the stock of capital falls. Therefore, during the recovery two years following the crisis the marginal product of capital and the return on capital are above where they otherwise would be. To see this effect observe that in the figure below (4.6) the rental rate of capital is above its non-QE path, and larger QE programs increase the magnitude of this difference.

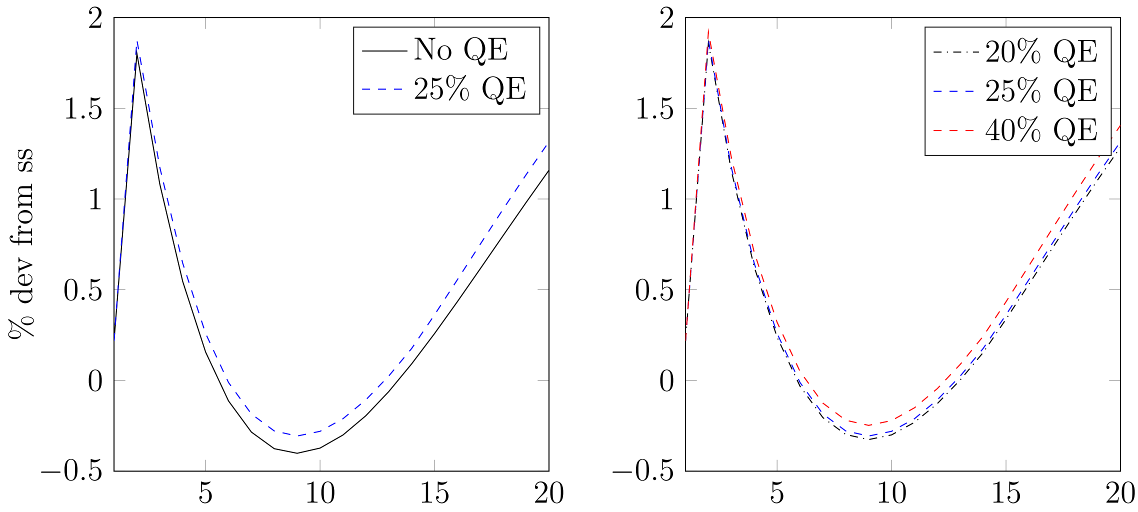

Figure 4.6: Rental Rate of Capital

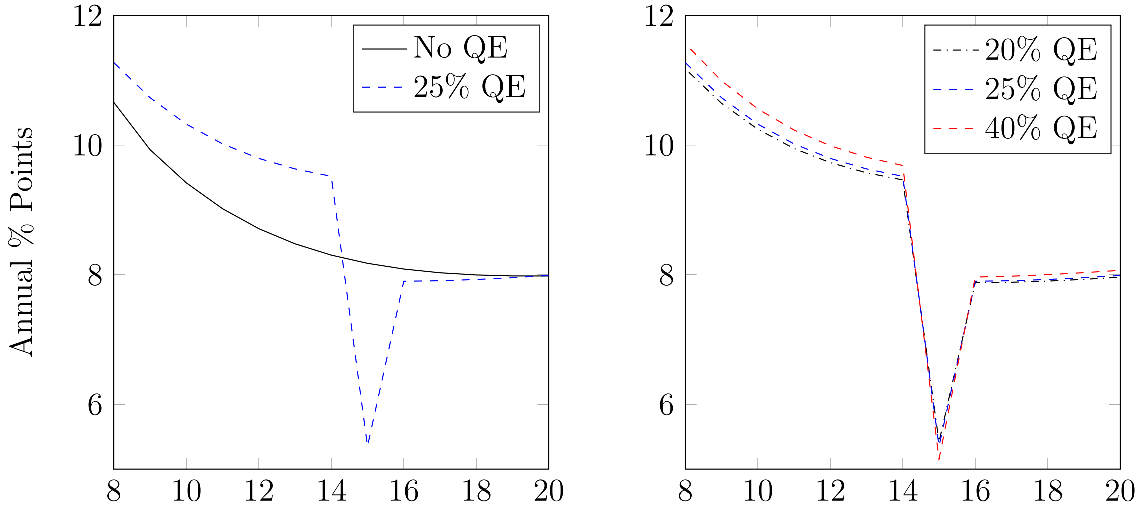

Figure 4.7: Return on Capital Starting Two Years After the Crisis

This means producers demand more capital from entrepreneurs. This increases the return on capital entrepreneurs earn, as can be seen in figure 4.7 above. As a consequence they in turn demand more raw capital from households. Due to the adjustment cost on investment, households are sluggish in responding to this change in demand. This causes the price of capital under QE to temporarily boom above its non-QE level as the increase in demand for capital is reconciled with the slowly adjusting supply. See this effect in figure 4.8 below. As equation (2.35) indicates, an increase in the price of capital will have a positive effect on aggregate net worth. Also note that the variations in level of QE do not seem to have much of an effect on the price of capital. This indicates that most of this effect comes from the second component of the unconventional monetary policy package, the constraint on the short rate.

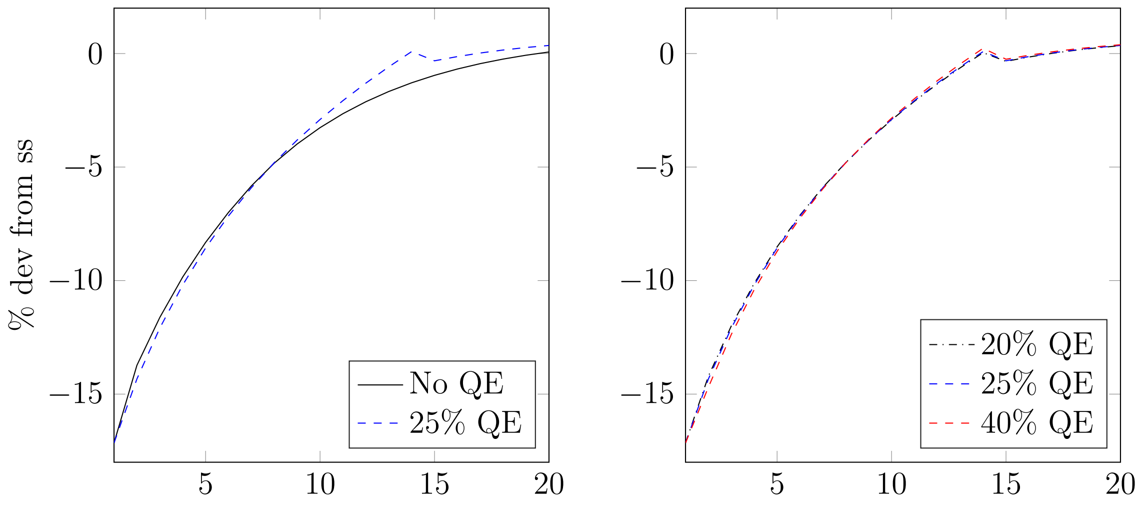

Figure 4.8: Price of Capital Two Years After the Crisis

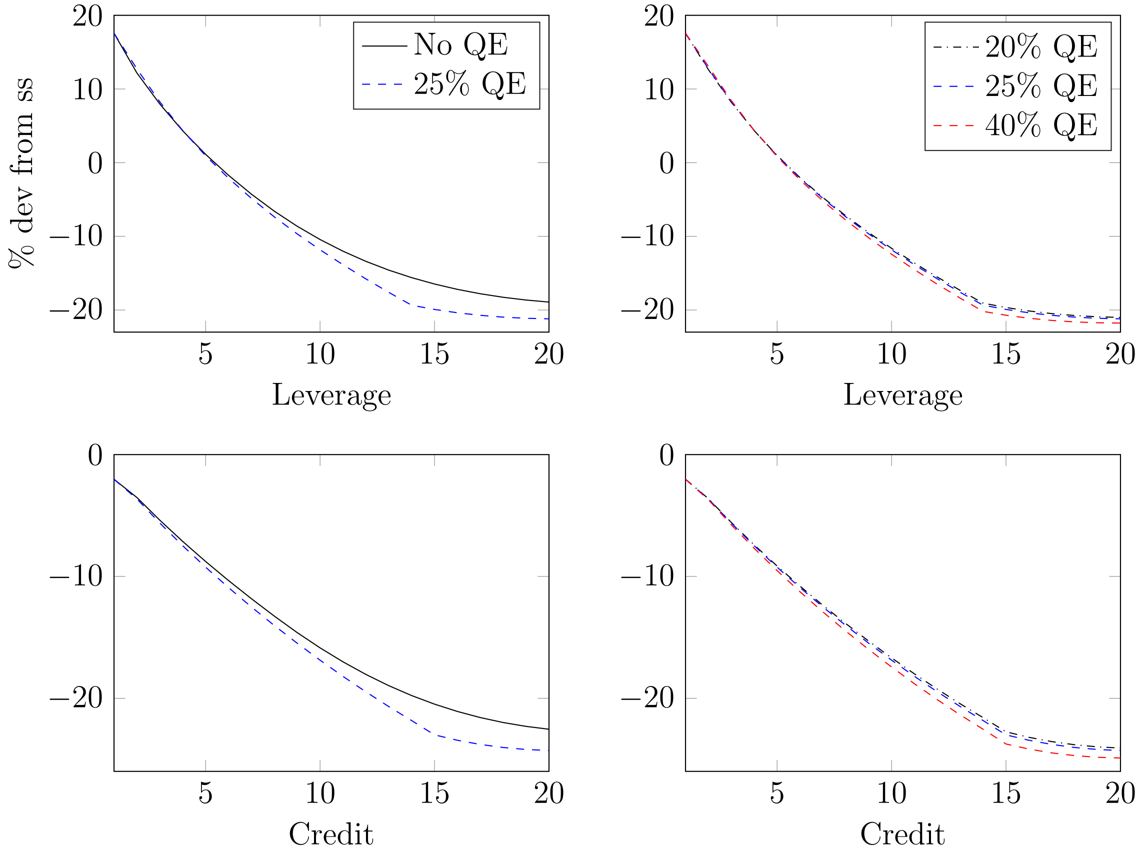

Figure 4.9: Leverage and Credit

The final element of the explanation as to why QE positively effects aggregate net worth involves the entrepreneur's choice of debt contract. As can be seen in figure 4.9 above, QE lowers the paths of leverage, $L_t \equiv \frac{Q_{\bar{K},t}\bar{K}_{t+1}}{N_{t+1}}$, and aggregate credit, $B^{credt}_{t+1} \equiv Q_{\bar{K},t}\bar{K}_{t+1} - N_{t+1}$. At first glance it is puzzling, given the higher return on capital under QE, why entrepreneurs do not increase their leverage to take advantage of the higher return. This is because QE's impact on the price of capital increases the value of entrepreneurial net worth. Entrepreneurs have more "inside funds" (their own net worth) to use for investment. So they use QE's initial positive impact on their net worth via the price of capital as an opportunity to substitute away from expensive loans from mutual funds to cheaper inside financing. Thus both credit and leverage are lower under QE.

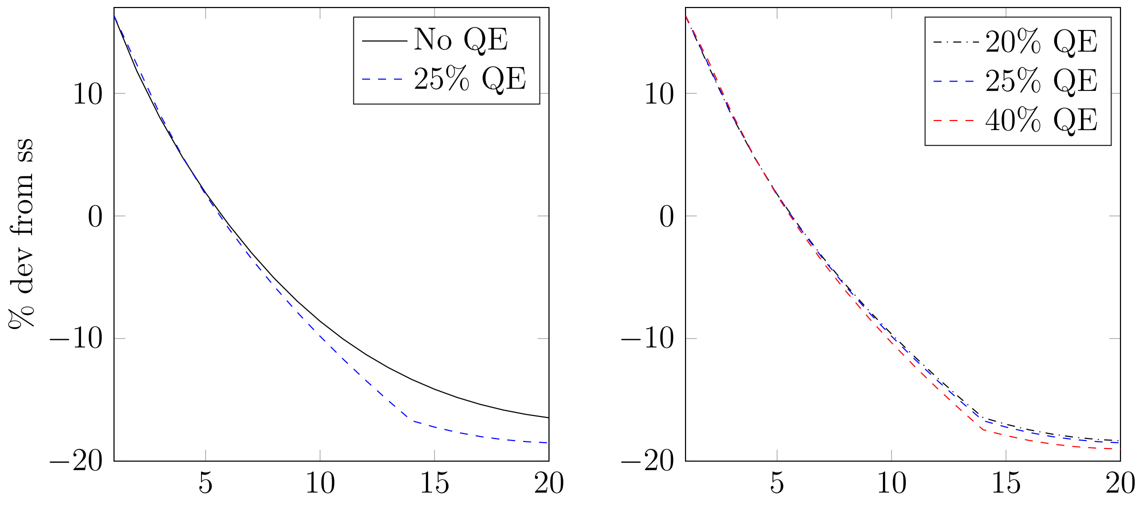

Figure 4.10: Entrepreneur Chosen Default Threshold

QE lowers the path of the second element of the debt contract, the default threshold $\bar{\omega}_t$ (see figure 4.10 above). This is not the result of a direct choice over $\bar{\omega}_t$ by entrepreneurs. Instead it comes from the effect of the sustained ZLB commitment on the mutual funds' cash constraint (2.28). R_t, the rate mutual funds must pay on the deposits they take, is lower for longer under the QE package. After 8 periods the commitment to the ZLB expires in the crisis-only simulation, wereas it persists in the QE simulation. Therefore after 8 periods in the crisis-only simulation the mutual funds' cash constraint starts to become more binding, under QE it does not. In the crisis-only world the rate entrepreneurs pay on their loads (Z_{t+1}) starts to rise but in the QE only world it does not. Z_{t+1} determines $\bar{\omega}_t$, a lower Z_{t+1} under QE results in the observed lower path for $\bar{\omega}_t$. This lower path for the default threshold means fewer entrepreneurs default, leading to an increase in aggregate net worth.

In summary the impact of QE on net worth is accelerated via the debt contract, $(L_t, \bar{\omega}_t)$, because it reduces both the incidence of default and the proportion of solvent entrepreneurs' earnings paid to mutual funds.

Conclusion

Using a calibrated DSGE model with a financial accelerator mechanism this thesis breaks Wallace's Irrelevance Result in order to study QE bond purchases by introducing a household preference over the portfolio of bonds held, motivated by Tobin's concept of imperfect substitutability. The resulting simulations suggest that during a crisis the QE policy works via the following mechanism.

The impact QE has on inflation induces households to move funds from investment to consumption. This means the stock of capital is smaller, so that, as the economy begins to recover from the Crisis Shock, the marginal product of capital is above where it would otherwise be. Producers want more capital; so entrepreneurs want to buy more raw capital from households. Adjustment costs mean that households cannot immediately respond to this demand. Therefore the price of capital goes up, and because entrepreneurs are less leveraged under QE, so does aggregate net worth.

By using a model with a financial sector and financial frictions, this thesis is able to examine both the macroeconomic and financial market impacts of the unconventional monetary policy pursued by the Federal Reserve in the wake of the 2007-08 financial crisis. The findings, that the use of unconventional monetary policy amplify the post-crisis recovery in the stock market, are qualitatively in line with the most striking result of the Federal Reserve's use of unconventional policies: the boom in the US stock market.

Further work is necessary to establish, robustly, the quantitative size of these observed effects. By the use of Bayesian estimation techniques and incorporation of anticipated components for exogenous shocks, the model in this thesis can be used to provide a quantitative assessment of the financial market and macroeconomic effects of the Federal Reserve's Quantitative Easing policy.

Even as the model stands in its present form, it shows that the inclusion of the Bernanke, Gertler, and Gilchrist financial accelerator expands the theoretical role quantitative easing policies can play inside DSGE models. Thus echoing the sentiment in the title of this thesis: that the renewed focus on financial frictions in the macroeconomics literature is essential for examining the full scope of the impact of unconventional monetary policy actions.

Bibliography

Andrés, Javier, J David López-salido, and Edward Nelson, "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium ," 2004, *36* (4), 665–690.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, "The Financial Accelerator in a Quantitative Business Cycle Framework," in John B. Taylor and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 1C, Amsterdam: North-Holland, 1999, chapter 21, pp. 1341–1393.

Carlstrom, Charles T. and Timothy S. Fuerst, "Agency Costs, Net Worth and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 1997, *87* (5), 893–910.

Chen, Han, Vasco Curdia, and Andrea Ferrero, "The Macroeconomic Effects of Large-Scale Asset Purchase Programmes," *Economic Journal*, 2012, *122* (November), F289–F315.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno, "Risk Shocks," *American Economic Review*, 2014, *104* (1), 27–65.

Del Negro, Marco, Marc P. Giannoni, and Christina Patterson, "The Forward Guidance Puzzle," *Federal Reserve Bank of New York Staff Reports 574*, 2013.

Dynare Team, "Dynare simult_," *Available at: http://www.dynare.org/dynare-matlab-m2html/matlab/simult_.html*, 2005.

Ellison, Martin and Andreas Tischbirek, "Unconventional government debt purchases as a supplement to conventional monetary policy," *Journal of Economic Dynamics and Control*, 2014, *43*, 199–217.

Eusepi, Stefano and Bruce Preston, "The Maturity Structure of Debt, Monetary Policy and Expectations Stabilization," *Working Paper, Federal Reserve Bank of New York*, 2011.

Federal Reserve Bank of St. Louis, "The Financial Crisis: Full Timeline," *Available at: www.stlouisfed.org/financial-crisis/full-timeline*, 2016.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, "Large-Scale Asset Purchases By the Federal Reserve: Did They Work?," *FRBNY Economic Policy Review*, 2011, (May), 41–59.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 2015, *70*, 22–38.

Harrison, Richard, "Asset Purchase Policy at the Effective Lower Bound for Interest Rates," *Bank of England, Working Paper No. 444*, 2012.

Modigliani, Franco and Richard Sutch, "Innovations in Interest Rate Policy," *American Economic Review*, 1966, *56* (1), 178–197.

Tobin, James and William C Brainard, "Financial Intermediaries and the Effectiveness of Monetary Controls," *American Economic Review*, 1963, *53* (2), 383–400.

Vayanos, Dimitri and Jean-Luc Vila, "A Preferred-Habitat Model of the Term Structure of Interest Rates," *Working Paper, London School of Economics*, 2009.

Wallace, Neil, "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review*, 1981, *71* (3), 267–274.

Equilibrium & Steady State Solution

A.1 Characterization of the Equilibrium

The equilibrium in this model requires equilibrium in the bond market, goods market (intermediate and final), and labour market. The resource constraint, household's budget constraint, and mutual fund cash constraint must hold. Lastly, interest rates must be such that household are willing to hold the available portfolio of bonds.

Lowercase variables are real detrended variables. So if X_t is a nominal variable then $x_t \equiv \frac{X_t}{P_t z_t^*}$.

Table A.1: Notation Key

$q_t \equiv \Upsilon^t \frac{Q_{\bar{K},t}}{P_t}$	$y_{z,t} \equiv \frac{Y_t}{z_t^*}$	$i_t \equiv \frac{I_t}{z_t^* \Upsilon^t}$
$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}$	$\bar{k}_t \equiv \frac{\bar{K}_t}{z_{t-1}^* \Upsilon^{t-1}}$	$\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$
$c_t \equiv \frac{C_t}{z_t^*}$	$b_t \equiv \frac{B_t}{z_t^* P_t}$	$b_t^L \equiv \frac{B_t^L}{z_t^* P_t}$
$g_t \equiv \frac{G_t}{z_t^*}$	$t_t \equiv \frac{T_t}{z_t^* P_t}$	$n_{t+1} \equiv \frac{N_{t+1}}{z_t^* P_t}$
$b_{t+1}^{credit} \equiv \frac{B_{t+1}^{credit}}{z_t^* P_t}$		

Equation 1 (First order condition with respect to consumption):

$$E_t\left\{\frac{\zeta_{c,t}\mu^*_{z,t}}{c_t\mu^*_{z,t}-bc_{t-1}}-\frac{\zeta_{c,t+1}b\beta}{c_{t+1}\mu^*_{z,t+1}-bc_t}-\lambda_{z,t}\zeta_{c,t}\right\}=0 \tag{A.1}$$

Equation 2 (First order condition with respect to the short bond):

$$E_t\left\{-\zeta_{c,t}\frac{\nu}{b_t^L}\left(\frac{b_t}{b_t^L}-1\right)-\lambda_{z,t}\zeta_{c,t}+\beta\frac{\lambda_{z,t+1}\zeta_{c,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\}=0 \tag{A.2}$$

Equation 3 (First order condition with respect to the long bond):

$$E_t\left\{\zeta_{c,t}\nu\frac{b_t}{(b_t^L)^2}\left(\frac{b_t}{b_t^L}-1\right)-\lambda_{z,t}\zeta_{c,t}+\beta\frac{\lambda_{z,t+1}\zeta_{c,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\right\}=0 \tag{A.3}$$

Equation 4 (First order condition with respect to investment):

$$E_t\left\{\zeta_{c,t}\lambda_{z,t}\left(q_t-\frac{1}{\mu_{\Upsilon,t}}\right)-\zeta_{c,t}\lambda_{z,t}q_t\left[S\left(\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right)+S'\left(\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right)\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right]\right.$$

(A.4)

$$\left.+\beta\frac{\lambda_{z,t+1}\zeta_{c,t+1}q_{t+1}}{\mu^*_{z,t}\Upsilon}S'\left(\frac{\zeta_{i,t+1}\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)\left(\frac{\zeta_{i,t+1}\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)^2\right\}=0$$

Equation 5 (Firm's Production Function):

$$y_{z,t}=(p_t^*)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}\left[\epsilon_t\left(\frac{u_t\bar{k}_t}{\mu^*_{z,t}\Upsilon}\right)^{\alpha}\left(h_t(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\right)^{1-\alpha}-\phi\right] \tag{A.5}$$

Equation 6 (Resource Constraint):

$$y_{z,t}=c_t+g_t+\frac{i_t}{\mu_{\Upsilon,t}}+\Theta\frac{1-\gamma_t}{\gamma_t}\left(n_{t+1}-w^e\right)+d_t+\frac{a(u_t)\bar{k}_t}{\Upsilon\mu^*_{z,t}} \tag{A.6}$$

Where $d_t\equiv\frac{\mu G_{t-1}(\bar{\omega}_t)R_t^k q_{t-1}\bar{k}_t}{\pi_t\mu^*_{z,t}}$

Equation 7 (Rental Rate of Capital from firm's profit maximization):

$$r_t^k=\alpha\epsilon_t\left(\frac{\mu^*_{z,t}\Upsilon h_t(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}}{u_t\bar{k}_t}\right)^{1-\alpha}s_t \tag{A.7}$$

Equation 8 (Marginal Cost):

$$s_t = \frac{1}{\epsilon_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} \tag{A.8}$$

Equation 9 (Capital Utilization Choice of Entrepreneurs):

$$r_t^k = a'(u_t) = r^k \exp(\sigma_a(u_t - 1)) \tag{A.9}$$

Where $a(u_t) \equiv \frac{r^k}{\sigma_a}\big[\exp(\sigma_a(u_t-1)) - 1\big]$.

Equation 10 (Rule for the Evolution of Capital):

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z,t}^{*}\Upsilon}\bar{k}_t + \Big[1 - S\Big(\frac{\zeta_{i,t} i_t \mu_{z,t}^{*}\Upsilon}{i_{t-1}}\Big)\Big] i_t \tag{A.10}$$

Equation 11 (Entrepreneur's gross rate of return on capital):

$$R_t^k = \frac{u_t r_t^k - a(u_t) + (1-\delta) q_t}{\Upsilon q_{t-1}}\pi_t \tag{A.11}$$

Equation 12 (Result of Entrepreneurs choosing the default level $\bar{\omega}_{t+1}$ to maximize their expected net worth subject to the Mutual Fund's Cash Constraint):

$$\begin{aligned} &E_t\Bigg\{\left[1 - \Gamma_t(\bar{\omega}_{t+1})\right]\frac{R_{t+1}^k}{R_t} \\ &+ \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})}\left[\frac{R_{t+1}^k}{R_t}\Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}\Big) - 1\right]\Bigg\} = 0 \end{aligned} \tag{A.12}$$

Equation 13 (Evolution of Entrepreneurial Net Worth):

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z,t}^{*}}\Bigg\{R_t^k(1 - \mu G_{t-1}(\bar{\omega}_t)) - R_{t-1}\Bigg\}\bar{k}_t q_{t-1} + w^e + \gamma_t \frac{R_{t-1}}{\pi_t \mu_{z,t}^{*}} n_t \tag{A.13}$$

Equation 14 (Zero Profit Condition on Mutual Funds):

$$\frac{q_{t-1}\bar{k}_t}{n_t}\frac{R_t^k}{R_{t-1}}\Big[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\Big] - \frac{q_{t-1}\bar{k}_t}{n_t} + 1 = 0 \tag{A.14}$$

Equation 15 (Government's consolidated budget constraint):

$$b_t + b_t^L = \frac{R_{t-1}b_{t-1}}{\pi_t \mu_{z,t}^*} + \frac{R_t^L b_{t-1}^L}{\pi_t \mu_{z,t}^*} + g_t - t_t \tag{A.15}$$

Equation 16 (Fiscal Feedback Rule):

$$t_t - g_t = \Phi_{tax}\left\{\frac{R_t^L b_{t-1}^L + R_{t-1}b_{t-1}}{R^L b^L + Rb}\frac{\pi \mu_z^*}{\pi_t \mu_{z,t}^*}\right\}^{\phi_{tax}} e^{\epsilon_t^{tax}} \tag{A.16}$$

Equation 17 (AR(1) for the supply of long term bonds):

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{BL}\log\left(\frac{b_{t-1}^L}{b^L}\right) - \epsilon_{BL,t} \tag{A.17}$$

Equation 18 (Taylor Rule):

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + \\ (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{y_{z,t}}{y_z} - \log\frac{\mu_z^*}{\mu_{z,t}^*}\right)\right] + \frac{1}{400}\epsilon_t^m \tag{A.18}$$

Equation 19 (Exogenous evolution of government spending):

$$\log\left(\frac{g_t}{g}\right) = \rho_g \log\left(\frac{g_{t-1}}{g}\right) + \epsilon_{g,t} \tag{A.19}$$

Where $g \equiv \eta_g y_z$.

Equation 20 (Shock equation for ϵ_t):

$$\log\left(\frac{\epsilon_t}{\epsilon}\right) = \rho_\epsilon \log\left(\frac{\epsilon_{t-1}}{\epsilon}\right) + \epsilon_{\epsilon,t} \tag{A.20}$$

Equation 21 (Shock equation for $\lambda_{f,t}$):

$$\log\left(\frac{\lambda_{f,t}}{\lambda_f}\right) = \rho_{\lambda_f}\log\left(\frac{\lambda_{f,t-1}}{\lambda_f}\right) + \epsilon_{\lambda_f,t} \tag{A.21}$$

Equation 22 (Shock equation for γ_t):

$$\log\left(\frac{\gamma_t}{\gamma}\right) = \rho_\gamma \log\left(\frac{\gamma_{t-1}}{\gamma}\right) + \epsilon_{\gamma,t} \tag{A.22}$$

Equation 23 (Shock equation for $\mu_{\Upsilon,t}$):

$$\log\left(\frac{\mu_{\Upsilon,t}}{\mu_\Upsilon}\right) = \rho_{\mu_\Upsilon} \log\left(\frac{\mu_{\Upsilon,t-1}}{\mu_\Upsilon}\right) + \epsilon_{\mu_\Upsilon,t} \tag{A.23}$$

Equation 24 (Shock equation for $\mu^*_{z,t}$):

$$\log\left(\frac{\mu^*_{z,t}}{\mu^*_z}\right) = \rho_{\mu^*_z} \log\left(\frac{\mu^*_{z,t-1}}{\mu^*_z}\right) + \epsilon_{\mu^*_z,t} \tag{A.24}$$

Equation 25 (Shock equation for π_t^{target}):

$$\log\left(\frac{\pi_t^{target}}{\pi^{target}}\right) = \rho_{\pi^{target}} \log\left(\frac{\pi_{t-1}^{target}}{\pi^{target}}\right) + \epsilon_{\pi^{target},t} \tag{A.25}$$

Equation 26 (Shock equation for σ_t):

$$\log\left(\frac{\sigma_t}{\sigma}\right) = \rho_\sigma \left(\frac{\sigma_{t-1}}{\sigma}\right) + \epsilon_{\sigma,t} \tag{A.26}$$

Equation 27 (Shock equation for $\zeta_{c,t}$):

$$\log\left(\frac{\zeta_{c,t}}{\zeta_c}\right) = \rho_{\zeta_c} \left(\frac{\zeta_{c,t-1}}{\zeta_c}\right) + \epsilon_{\zeta_c,t} \tag{A.27}$$

Equation 28 (Shock equation for $\zeta_{i,t}$):

$$\log\left(\frac{\zeta_{i,t}}{\zeta_i}\right) = \rho_{\zeta_i} \left(\frac{\zeta_{i,t-1}}{\zeta_i}\right) + \epsilon_{\zeta_i,t} \tag{A.28}$$

Equation 29:

$$\phi = steady\_state(\phi) \tag{A.29}$$

Equations Related to Price Setting:

Equation 30 (Law of motion for p_t^*):

$$p_t^* = \left[(1-\xi_p)\Big(\frac{K_{p,t}}{F_{p,t}}\Big)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} + \xi_p\Big(\frac{\tilde{\pi}_t}{\pi_t}p_{t-1}^*\Big)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}\right]^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}} \tag{A.30}$$

Equation 31 (Law of motion for $F_{p,t}$, relates to Calvo Frictions):

$$F_{p,t} = E_t\left\{\zeta_{c,t}\lambda_{z,t}y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}\beta\xi_p F_{p,t+1}\right\} \tag{A.31}$$

Equation 32 (Law of motion for $K_{p,t}$, which is defined in the auxiliary expressions):

$$K_{p,t} = E_t\left\{\zeta_{c,t}\lambda_{z,t}\lambda_{f,t}y_{z,t}s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}K_{p,t+1}\right\} \tag{A.32}$$

Equation 33 (Law of motion for $F_{w,t}$, characterizes optimal wage setting):

$$F_{w,t} = E_t\left\{\zeta_{c,t}\lambda_{z,t}(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\frac{h_t}{\lambda_w} + \beta\xi_w(\mu_z^*)^{\frac{1-\iota_\mu}{1-\lambda_w}}(\mu_{z,t+1}^*)^{\frac{\iota_\mu}{1-\lambda_w}-1}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}}F_{w,t+1}\right\} \tag{A.33}$$

Equation 34 (Law of motion for $K_{w,t}$):

$$K_{w,t} = E_t\left\{\zeta_{c,t}\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}h_t\right]^{1+\sigma_L} + \beta\xi_w K_{w,t+1}\left(\frac{\tilde{\pi}_{w,t+1}(\mu_{z,t+1}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}\right\} \tag{A.34}$$

Equation 35 (Law of motion for w_t^*):

$$w_t^* = \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}(\mu_z^*)^{1-\iota_\mu}(\mu_{z,t}^*)^{\iota_\mu}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}(\mu_{z,t}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t}}w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \tag{A.35}$$

Equation 36 (definition of credit):

$$B_{t+1}^{credit} = q_t \bar{k}_{t+1} - n_{t+1} \tag{A.36}$$

Equation 37 (definition of leverage, L_t):

$$L_t = \frac{1}{1 - \frac{R_{t+1}^k}{R_t}\Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\Big)} \tag{A.37}$$

<u>The Distributions</u>:

$$\Gamma_t(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1}\Big[1 - F_t(\bar{\omega}_{t+1})\Big] + G_t(\bar{\omega}_{t+1}) \tag{A.38}$$

$$\Gamma_t'(\bar{\omega}_{t+1}) \equiv 1 - F_t(\bar{\omega}_{t+1}) \tag{A.39}$$

$$F_t(\bar{\omega}_{t+1}) \equiv \Phi\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t}\right) \tag{A.40}$$

$$G_t(\bar{\omega}_{t+1}) \equiv \Phi\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t\right) \tag{A.41}$$

$$G_t'(\bar{\omega}_{t+1}) \equiv PDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t}\right)\frac{1}{\sigma_t} \tag{A.42}$$

<u>Auxilary Variables Not Defined Before</u>:

Definition of $K_{p,t}$:

$$K_{p,t} = F_{p,t}\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_{f,t}}}}{1-\xi_p}\right]^{1-\lambda_{f,t}} \tag{A.43}$$

Definition of $K_{w,t}$:

$$K_{w,t} \equiv \frac{\tilde{w}_t F_{w,t}}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}(\mu_z^*)^{\iota_\mu}(\mu_{z,t}^*)^{1-\iota_\mu}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(\sigma_L+1)} \tag{A.44}$$

Definition of Wage Inflation $\pi_{w,t}$:

$$\pi_{w,t} \equiv \pi_t \mu_{z,t}^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \tag{A.45}$$

A.2 Steady-State Solution Strategy

In the steady state all parameters are fixed to their calibrated values. With the exception of the following four parameters that are calibrated to match certain values for endogenous variables in steady state.

- β: calibrate to make R consistent with the data (equations 2 and 3).
- ψ_L, calibrate so that labour hours h is equal to unity in steady state.
- ν: calibrate so that equations 2 and 3 imply the average term premium in the data: $R^L - R = 0.0042$.
- $\Phi_{tax} \equiv y_z\Big\{0.4113\frac{R}{\pi\mu_z^*} + 0.7627\frac{R^L}{\pi\mu_z^*} - (0.4113 + 0.7627)\Big\}$.

Where the calibration of Φ_T comes from the fact that the steady state quantity of short and long bonds as a fraction of steady-state output is 0.4113 and 0.7627 respectively.

Step 1: Equations 20, 21, 23-25, 27, and 28 gives the steady state values of the following variables. $\epsilon, \lambda_f, \mu_\Upsilon, \mu_z^*, \pi^{target}, \zeta_c, \zeta_i$.

Step 2: Equation 18 (The Taylor Rule) implies that $\pi = \pi^{target}$.

Step 3: In steady state $S(\frac{\zeta_i \mu_z^* \Upsilon i}{i}) = S'(\frac{\zeta_i \mu_z^* \Upsilon i}{i}) = 0$ So Eqn 4, the household's FOC wrt investment becomes: $\zeta_c \lambda_z \Big(q - \frac{1}{\mu_\Upsilon}\Big) = 0$. Thus the scaled price of capital in steady state $q = \frac{1}{\mu_\Upsilon}$.

Step 4: Equation 9 in SS is $r^k = r^k \exp(\sigma_a(u-1))$, implies $u = 1$.

Step 5: Set $h = 1$ and later will find ψ_L so that this holds. Calibrate $R^L - R, R^k - R$, and R from the data. This gives $R^L - R = 0.0042$, and $R^k - R = 0.0073724$. This spread between the prime lending rate and the 3 month US Treasury corresponds to 2.98% annualized. This is similar to the spread in Carlstrom and Fuerst (1997) which targeted a $\approx$ 300 bps spread.

Step 6: Target $F(\omega) = 0.0056$, which corresponds to a 2.24% annualized entrepreneur failure rate, and use equation 12 in steady state to solve for σ and $\bar{\omega}$. Use the results to define the steady-state distributions $F(\omega), G(\omega), G'(\omega), \Gamma(\omega)$, and $\Gamma'(\omega)$.

Step 7: Use equation 11 in steady state: $R^k = \pi \frac{(ur^k - a(u)) + (1-\delta)q}{\Upsilon q}$. $a(u) = 0$. Solve to get r^k in steady state:
$r^k = \frac{1}{u}\Bigg[\frac{R^k \Upsilon q}{\pi} - (1-\delta)q\Bigg]$.

Step 8: Define the Auxiliary Variables in steady state:

$$\tilde{\pi} = (\pi^{target})^{\iota}(\pi)^{1-\iota} \tag{A.46}$$

$$\tilde{\pi}_w = (\pi^{target})^{\iota_w}(\pi)^{1-\iota_w} \tag{A.47}$$

$$\pi_w = \pi \mu_z^* \tag{A.48}$$

Step 9: The definition of Kp gives ratio between K_p and F_p in steady state:

$$\frac{K_p}{F_p} = \left[\frac{1-\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{1-\lambda_f} \tag{A.49}$$

Step 10: Equation 30, the law of motion for p^*, gives the following expression for p^* in steady state.

$$p^* = \left\{\frac{(1-\xi_p)\left(\frac{K_p}{F_p}\right)^{\frac{\lambda_f}{1-\lambda_f}}}{1-\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}}\right\}^{\frac{1-\lambda_f}{\lambda_f}} \tag{A.50}$$

Step 11: Equation 31, the law of motion for F_p, solved for F_p gives:

$$F_p = \frac{\zeta_c \lambda_z y_z}{1-\beta\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}} \tag{A.51}$$

Eqn 32, the law of motion for K_p solved for F_p gives:

$$F_p = \left(\frac{K_p}{F_p}\right)^{-1} \frac{\zeta_c \lambda_z \lambda_f y_z s}{1-\beta\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}} \tag{A.52}$$

Equating the two expressions gives the following result for s in steady state:

$$s = \frac{K_p}{F_p}\frac{1}{\lambda_f}\frac{1-\beta\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}}{1-\beta\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}} \tag{A.53}$$

Step 12: Solve equation 8. the expression for marginal cost, in steady state to get the following expression for $\tilde{w}$

$$\tilde{w} = (1-\alpha)\left[s\epsilon\left(\frac{r^k}{\alpha}\right)^{-\alpha}\right]^{\frac{1}{1-\alpha}} \tag{A.54}$$

Step 13: Eqn 35, law of motion for w^* in steady state gives the following expression for w^*.

$$w^* = \left\{ \frac{(1-\xi_w)\left\{ \frac{1-\xi_w\left[\frac{\tilde{\pi}_w}{\pi_w}(\mu_z^*)^{1-\iota_\mu}(\mu_z^*)^{\iota_\mu}\right]^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right\}^{\lambda_w}}{1-\xi_w\left[\frac{\tilde{\pi}_w(\mu_z^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_w}\right]^{\frac{\lambda_w}{1-\lambda_w}}} \right\}^{\frac{1-\lambda_w}{\lambda_w}} \tag{A.55}$$

Step 14: Equation 34, law of motion for K_w, in steady state gives the following expression for K_w

$$K_w = \frac{\zeta_c\left((w^*)^{\frac{\lambda_w}{\lambda_w-1}}h\right)^{(1+\sigma_L)}}{1-\beta\xi_w\left[\frac{\tilde{\pi}_w}{\pi_w}(\mu_z^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}\right]^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}} \tag{A.56}$$

Step 15: Equation 7, rental rate on capital, in steady state implies the following for the ratio of h to $\bar{k}$

$$\frac{h}{\bar{k}} = \left(\frac{r^k}{s\alpha\epsilon}\right)^{\frac{1}{1-\alpha}}\frac{(w^*)^{\frac{\lambda_w}{1-\lambda_w}}}{\Upsilon\mu_z^*} \tag{A.57}$$

Then, because h is known, solve for $\bar{k} = h\left(\frac{h}{\bar{k}}\right)^{-1}$.

Step 16: Equation 10 (the law of motion for capital) in steady state

$$i = \bar{k}\left[1-\frac{1-\delta}{\mu_z^*\Upsilon}\right] \tag{A.58}$$

Step 17: Then ϕ can be computed to guarantee zero profits in steady-state:

$$\phi = \left(\frac{\bar{k}}{\mu_z^*\Upsilon}\right)^{\alpha}h^{1-\alpha}\left(1-\frac{1}{\lambda_f}\right) \tag{A.59}$$

Step 18: Equation 5, output, gives y_z:

$$y_z = (p^*)^{\frac{\lambda_f}{\lambda_f-1}}\left[\epsilon\left(\frac{u\bar{k}}{\mu_z^*\Upsilon}\right)\left((w^*)^{\frac{\lambda_w}{\lambda_w-1}}h\right)^{1-\alpha} - \phi\right] \tag{A.60}$$

Step 19: Use equation 19, 17, 15, & 16 to find g, b^L, b, Φ_{tax}, and t, which are all proportional to y_z.

Step 20: Use equation 14, the mutual fund's zero profit condition, to get n

$$n = \bar{k}\left[1 - \frac{R}{R^L}\big(\Gamma(\omega) - \mu G(\omega)\big)\right] \tag{A.61}$$

Step 21: Use equation 13 (the law of motion for entrepreneurial net worth) in steady state to get γ

Step 22: Use equation 6 (the resource constraint), to find c in steady state

$$c = y_z - g - \frac{i}{\mu_\Upsilon} - \Theta\frac{1-\gamma}{\gamma}(n - w^e) - d \tag{A.62}$$

Where $d \equiv \frac{\mu G(\omega) R^k q \bar{k}}{\pi \mu_z^*}$

Step 23: Use equations 1-3 in steady state to get λ, β, and ν.

Step 24: Equation 33, law of motion for F_w in steady state implies the following for F_w.

$$F_w = \frac{\zeta_c \lambda_z (w^*)^{\frac{\lambda_w}{\lambda_w - 1}} \frac{h}{\lambda_w}}{1 - \beta\xi_w (\mu_z^*)^{\frac{1-\iota_\mu}{1-\lambda_w}} (\mu_z^*)^{\frac{\iota_\mu}{1-\lambda_w} - 1} \left(\frac{1}{\pi_w}\right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_w^{\frac{1}{1-\lambda_w}}}{\pi}} \tag{A.63}$$

Step 25: Eqn 31, law of motion for F_p, in steady state gives the following expression for F_p

$$F_p = \frac{\zeta_c \lambda_z y_z}{1 - \beta\xi_p \left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}} \tag{A.64}$$

Step 26: The definition of K_w in steady state gives the calibration for ψ_L.

$$\psi_L \equiv \tilde{w}\frac{F_w}{K_w}\left\{\frac{1 - \xi_w\left[\frac{\tilde{\pi}_w}{\pi_w}\mu_z^*\right]^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right\}^{1-\lambda_w(1+\sigma_L)} \tag{A.65}$$

Note: The nested CMR model's steady state can be found via the same method. Simply set the term premium equal to zero instead of 0.0042. And adjust the steady state specific calibrated parameters appropriately.

Collected Results Appendix

B.1 Nested CMR 10% Risk Shock

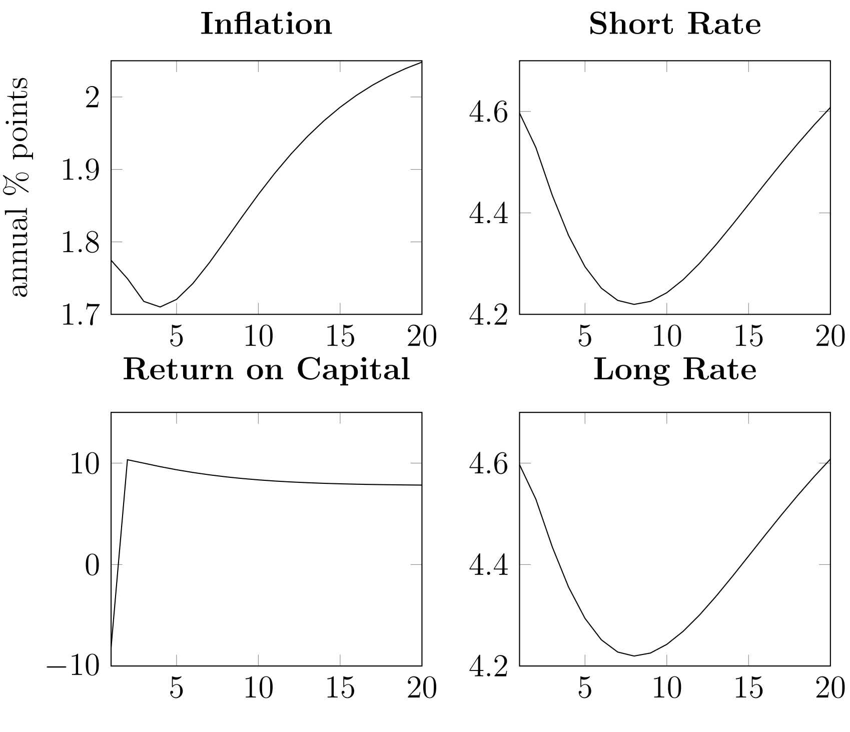

Figure B.1: 10% Risk Shock in Nested CMR

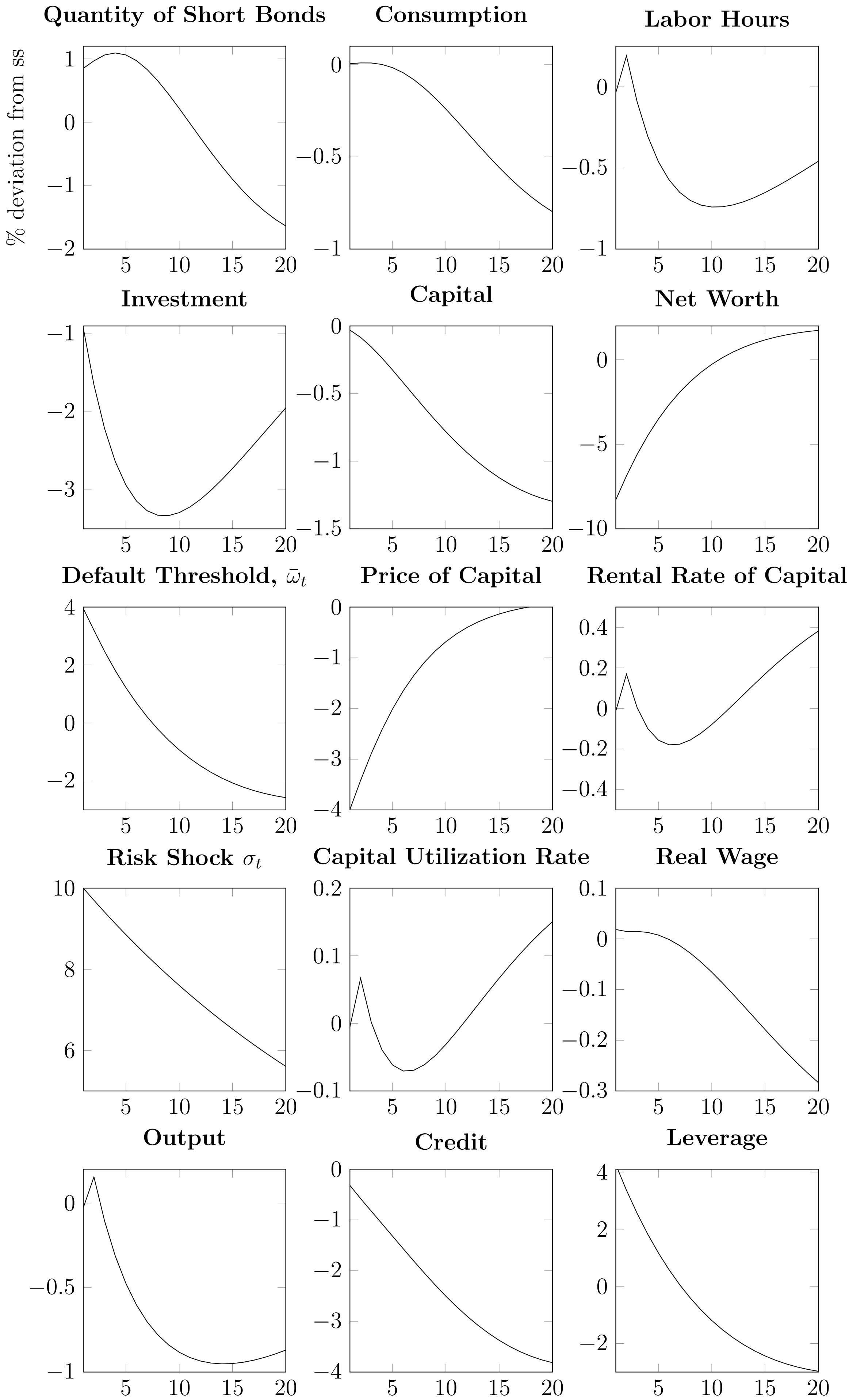

Figure B.2: 10% Risk Shock in Nested CMR

B.2 Liquidity Preference Model 10% Risk Shock

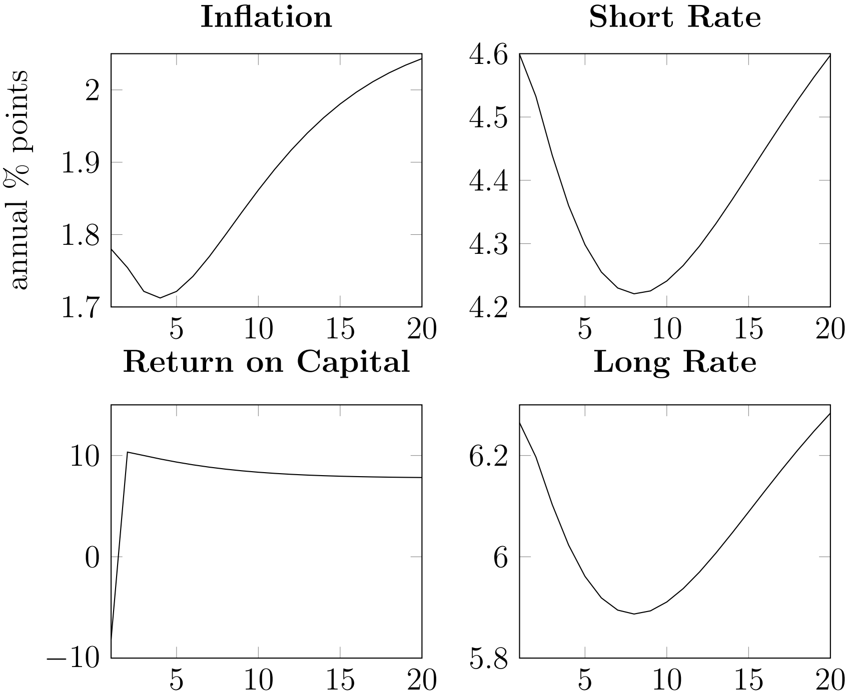

Figure B.3: 10% Risk Shock in LP Model

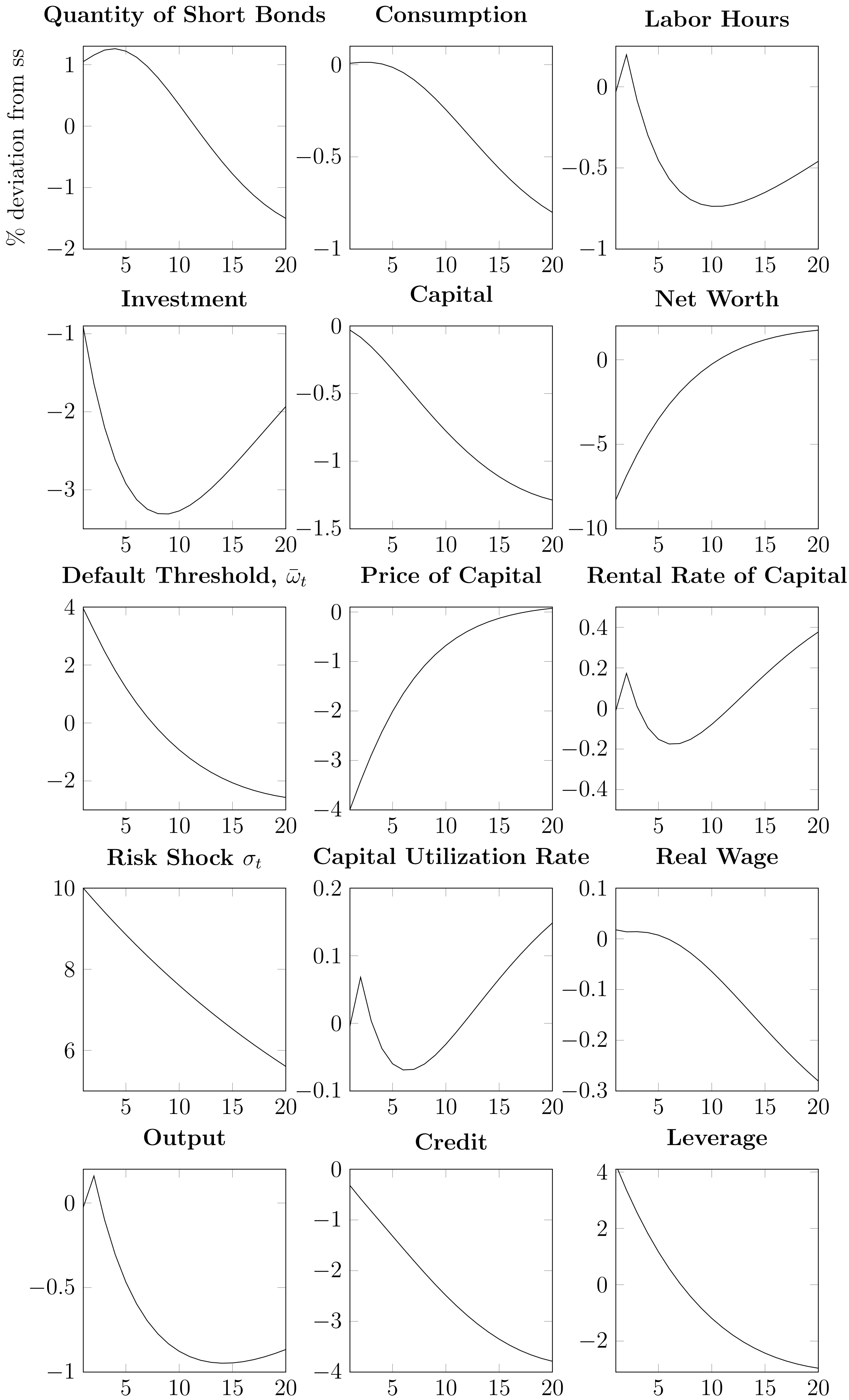

Figure B.4: 10% Risk Shock in LP Model

B.3 Crisis with and without 25% QE and the ZLB Commitment

The following (black solid line) IRFs are for the crisis model's response to a 60% risk shock. Note that an expansionary monetary policy shock is applied in this simulation for the first two periods after the crisis risk shock hits. This ensures that the short rate immediately falls to the zero lower bound in the period after the crisis shock, thus maintaining consistency with the timeline. Without policy intervention the ZLB ceases to bind 8 periods after the crisis shock, without QE intervention, hits.

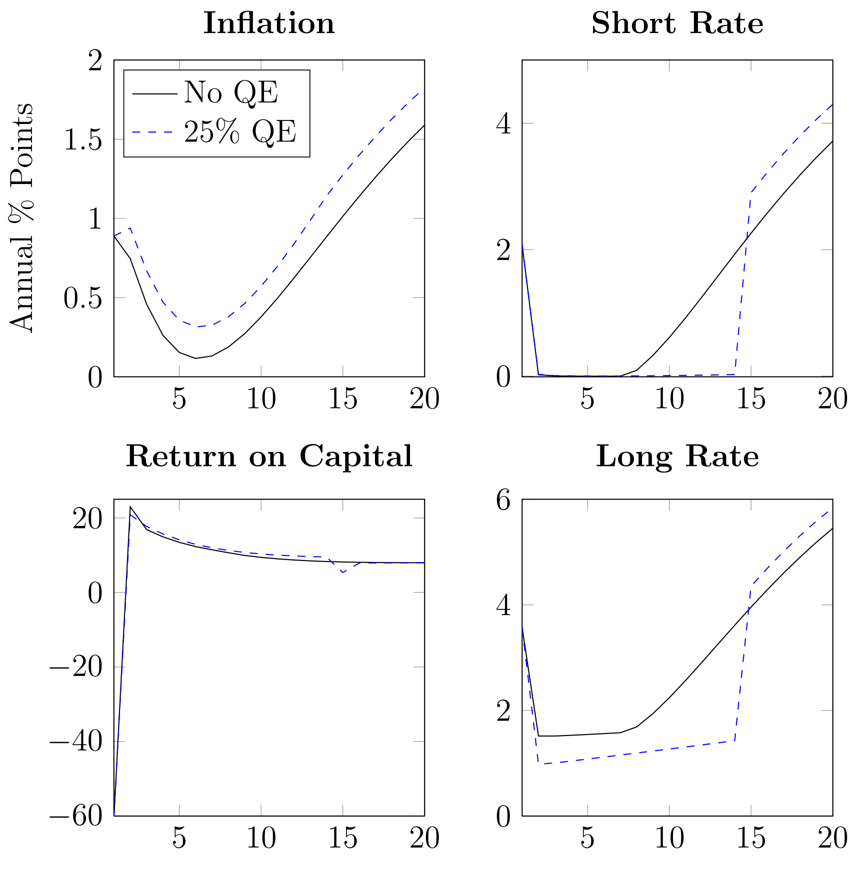

Figure B.5: Crisis vs 25% QE

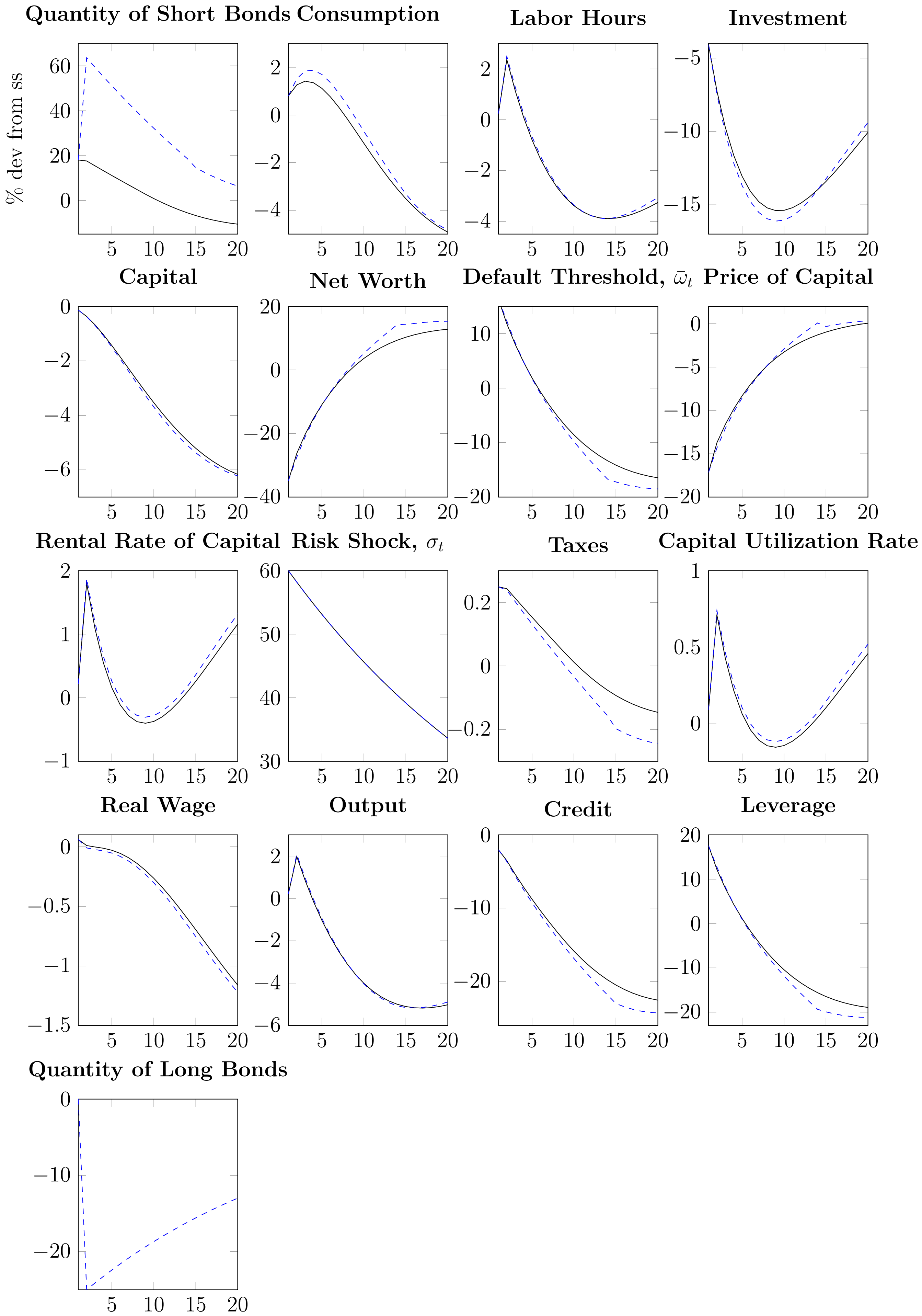

Figure B.6: Crisis vs 25% QE

B.4 Comparison Between Varying QE Policy Sizes

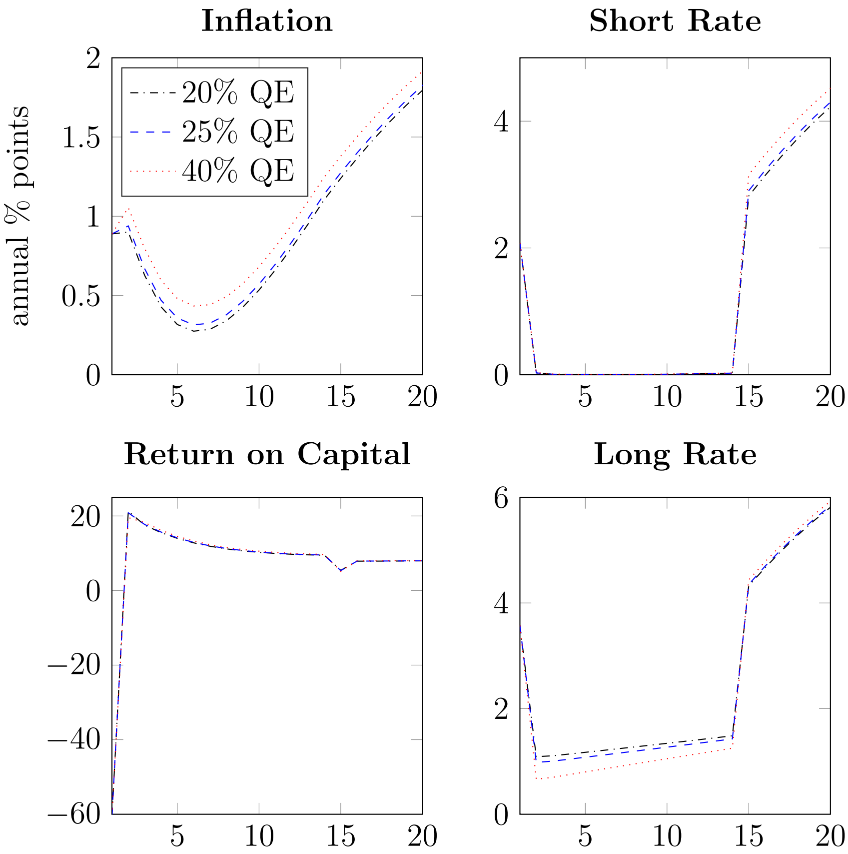

Figure B.7: Varying Levels of QE

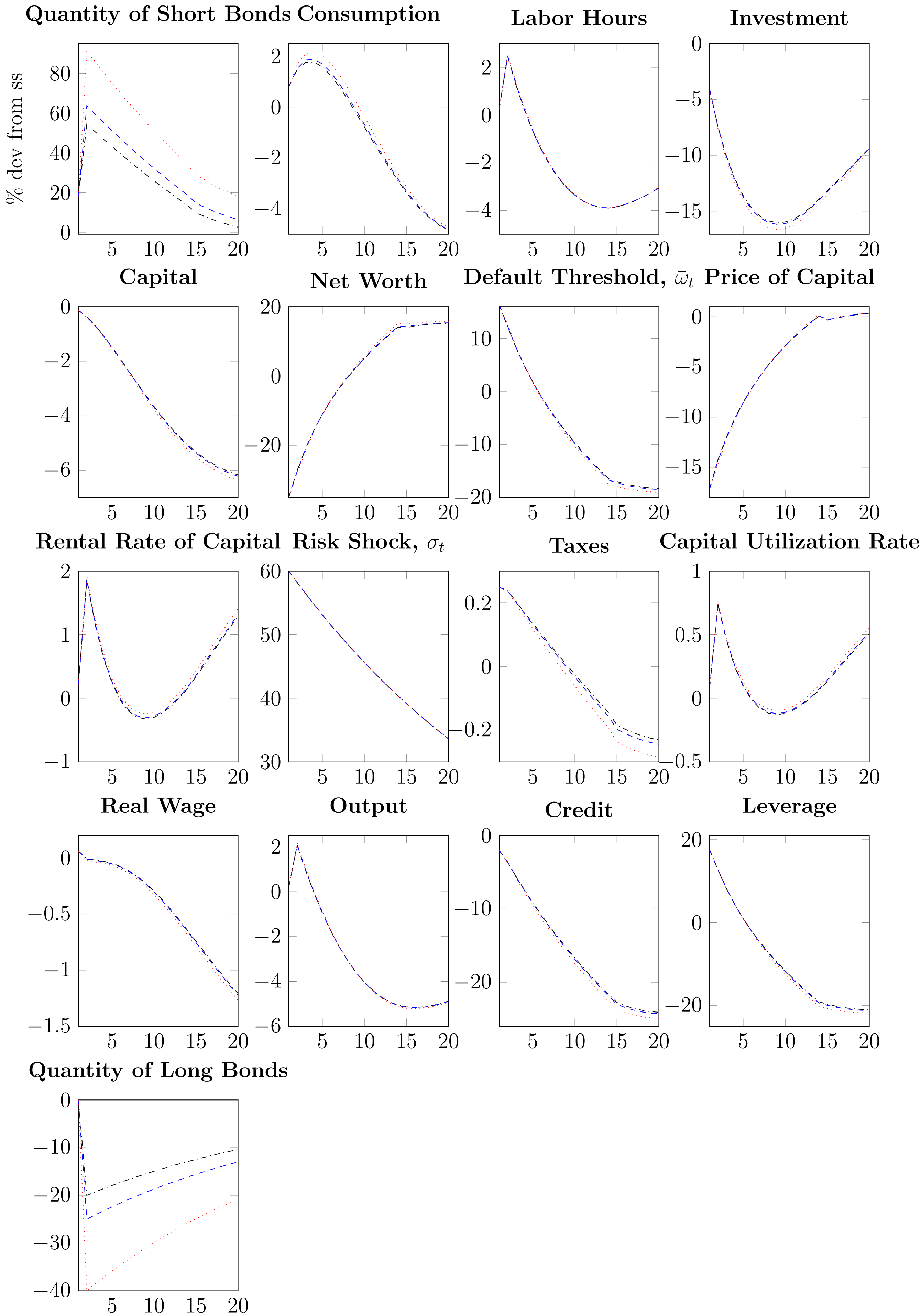

Figure B.8: Varying Levels of QE